A Comparative Study of Stochastic Models for Forecasting Electricity Generation and Consumption in Nigeria

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Abstract. With energy serious shortage of the Nigerian Power Sector owing to industry deregulation, abrupt variations in electricity demand, and increasing population density, Nigeria's economic development has been restricted. Thus, it is significant to balance the relationship between power generation and consumption, and further stabilize the two in a reasonable scope. To achieve balance, an accurate model to fit and predict electricity generation and consumption in Nigeria is required. This study, therefore, proposes a comparative study on stochastic modeling; (Harvey model, Autoregressive model, and Markov chain model) for forecasting electricity generation and consumption in Nigeria. The data gathered were analyzed and the model parameters were estimated using the maximum likelihood estimation technique. The comparative performance revealed that the Markov chain model best-predicted electricity generation than the Harvey and Autoregressive models. Also, for electricity consumption, results showed that the Harvey model predicted best than the Markov and Autoregressive models for electricity consumption. Thus, the Markov and Harvey model used to forecast electricity generation and consumption in Nigeria for the next 20 years (2018 to 2037) did not only reveal that electricity generation and consumption will continue to increase from 3,692.11 mln kW/h to 18,250.67 mln kW/h and from 2,961.10 mln kW/h to 127,071.30 mln kW/h respectively but also indicates high accuracy and the reference value of these models.

Keywords: Autoregressive model; Electricity generation; Electricity consumption; Forecasting; Harvey model; Markov model.

INTRODUCTION

The generation of electric power in Nigeria is overwhelmed by excessive demand for electricity by consumers because of inadequate supply. This supply shortfall has resulted in prolonged and intermittent power outages supplies to the consumers over the years. It is the belief that efficient power supply results in quality health care and economic growth on nation-building to mention a few [1]. Growth results in an increase in power demand, which certainly requires planning ahead of time to meet the present and future demand for uninterruptible power supply [2].

Forecasting electricity generation and consumption with high accuracy is important as it helps to plan production along with required demand in advance and prevent energy wastage and system failure. Electricity consumption forecasting is one of the most significant challenges in dealing with the supply and demand of electricity. Also, accurate forecast leads to increase the reliability of power supply, precise decision making for future development, quality savings in operation, and maintenance costs [3]. The dynamic nature of the electricity market, therefore, requires that an investor in power generation must be sure that there is a demand for electricity before setting up a generation plant, while distribution companies will want to be guaranteed that there is an available supply for their customers. Hence, a safe and reliable source of electricity involves a feasible and practical method for demand forecasting.

Many theoretical methods including growth curves, multiple linear regression methods that use economic, social, geographic, and demographic factors, and Box-Jenkins autoregressive moving average (ARMA) and autoregressive integrated moving average (ARIMA) techniques, Harvey logistic model, Harvey model, and Autoregressive model has been applied in forecasting electricity generation and consumption. Likewise, different research works have compared various models to determine which has a better forecasting accuracy. The task of ensuring power supply has become so important that researchers use various predictive models to conduct research and analysis on power in different countries. At the same time, the best estimate for the forecast of these predictive models is helpful for forecasting demand in other energy sectors. Nonetheless, there are not many shaping documents such as power prediction and accuracy comparison by using some models at the same time.

A study to determine the best model for forecasting the prices of electricity in a competitive market was shown by [4]. They compared four models; AR, MA, GARCH, and ARCH models to determine the best model and provide the estimates of electricity prices based on the best model. Other variables that provide energy in the industries were used to test the validity of the model. The models were ranked on the bases of the Akaike information criterion (AIC) and the Bayesian Information Criterion (BIC). The empirical analysis revealed that the ARMA (2,1,2) had the lowest root mean square error and the mean absolute percentage error than the GARCH (2, 1) model which indicates that the ARMA is a better model in forecasting the electricity prices than the GARCH model when there exist exogenous variables. Authors [5] used the GARCH model to estimate the volatility of the marketplace, while Harvey logistic model was used to forecast electricity demand and supply in Nigeria between 2005 and 2026. Authors [6] forecasted electricity demand in Tamale Ghana using the ARIMA model. Secondary data from 1990 to 2013 was applied, and the result showed that both domestic and commercial demand was increasing more rapidly than industrial sector demand. In [7] predicting electricity consumption using regression, the Kalman filter adaptation algorithm and ANN was investigated. Empirical results from the analysis showed that the Kalman filter adaptation algorithm was the best in terms of future prediction of electricity consumption. Authors [8] modeled and predicted residential electricity utilization in Nigeria using multiple/quadratic regression models. Empirical analysis showed that the quadratic regression model outperformed the multiple regression model. Authors [9] conducted a comparative study on medium-term load forecasting using Artificial Neural Network, (ANN) and regression model. Results showed that ANN-model performed better than the regression model for load forecasting. In [10], a study on long term electric load forecasting on the Nigerian power system using the modified form of the exponential regression model was carried out. The model was used to predict residential, commercial, and industrial load demand.

Authors [11] applied the Markov model in crude oil price forecasting. They found patterns in past crude oil price datasets that match with today's crude oil price behavior, then incorporate these two datasets with appropriate neighboring price elements to forecasting tomorrow's crude oil price. Based on a state sequence, three different states were assumed, with state-space $S = (S_1, S_{2,} S_3)$, S_1 =Up, S_2 =Same and S_3 =Down, which were decided by comparing the previous closing price and the current closing price. The number of days that both the first day and the second day are up to was calculated using data obtained on the closing index from WTI (West Texas Intermediate) for daily crude oil prices from 2nd January 2015 to 29th May 2015 to model the process. Results obtained showed that the transition matrix was stable, and the most likely trend of the index is down since the probability of down is the biggest. The previous price dated 29th May 2015 was $60.25 and the price of the predicted day, 1st June 2015 was $60.24 respectively. The result shows that forecasting is accurate and reliable. Thus, they concluded that the Markov model can produce an accurate forecast based on the description of historical patterns in crude oil prices.

In this research, three models; Harvey, Autoregressive, and Markov Chain Models will be compared on historical data of electricity generation and consumption in Nigeria and determine which of the three models has a better prediction accuracy. The model with the best fit will be used to forecast electricity generation and consumption for the next twenty years; (2017-2036).

METHODS

Autoregressive model. An autoregressive (AR) model predicts future outcomes based on the past outcome. In an AR model, the value of the outcome variable (Y) at some point t in time is directly related to the predictor variable (X). It is simply a linear regression of the current value of the series against one or more prior values of the series. The value of p is called the order of the AR model. AR models can be analyzed with one of the various methods, such as the standard linear least square techniques. A common approach for modeling univariate time series is the AR model:

$$X_t = \mu + \emptyset_1 X_{t-1} + \emptyset_2 X_{t-2} + \cdots + \emptyset_p X_{t-p} + \varepsilon_t$$

where X_t is the time series and ε_t is the white noise, with μ denoting the process mean.

An autoregressive model of order p, denoted by AR (p) with mean zero is generally given the equation:

$$X_t = \phi_1 X_{(t-1)} + \phi_2 X_{(t-2)} + \phi_3 X_{(t-3)} + \cdots + \phi_p X_{t-p} + \varepsilon_t$$

Or

$$X_t = (\phi_1 L + \phi_2 L^2 + \phi_3 L^3 + \cdots + \phi_p L^p) X_t + \varepsilon_t$$

where $\phi(L) = \varepsilon_t$

$$\phi(L) = (1 - \phi_1 L + \phi_2 L^2 + \phi_3 L^3 + \cdots + \phi_p L^p)$$

where L is the lag operator;

$\phi_1, \phi_2, \phi_3, \dots, \phi_p$ $(\phi_p \neq 0)$ are the autoregressive model parameters and ε_t is the random shock or white noise process, with mean zero and variance σ_ε^2. The mean of X_t is zero. If the mean, μ of X_t is not zero, replace X_t by $X_{t-\mu}$, i.e.

$$X_{t-\mu} = \phi_1 (X_{t-1} + \mu) + \phi_2 (X_{t-2} + \mu) + \cdots + \phi_p (X_{t-p} + \mu) + \varepsilon_t$$

Or write,

$$X_t = c + \phi_1 X_{t-1} + \phi_2 X_{t-2} + \cdots + \phi_p X_{t-p} + \varepsilon_t$$

where $c = \mu(1 - \phi_1 - \phi_3 \dots . \phi_p)$.

An AR (p) model is stationary if the roots of $\phi(L) = 0$ all lie outside the unit circle. A necessary condition for stationary is that $r_k = 0$ as $k \to \infty$.

Maximum likelihood estimation (MLE) for autoregressive models. Given an AR (1) model

$$x_t = c + \phi x_{t-1} + \varepsilon_t \quad (1)$$
$$\varepsilon_t \sim iid\ N(0, \sigma^2), t = 1, \dots, T$$
$$\theta = (c, \phi, \sigma^2)', |\phi| < 1$$

conditional on I_{t-1}

$$x_t | I_{t-1} \sim N(c + \phi x_{t-1}, \sigma^2), t = 2, \dots, T$$

which only depends on x_{t-1}. The conditional density $f(x_t | I_{t-1}, \theta)$ is then:

$$f(x_t | x_{t-1}, \theta) = (2\pi\sigma^2)^{-1/2} exp\left(-\frac{1}{2\sigma^2}(x_t = c + \phi x_{t-1})^2\right), t = 2, \dots, T \quad (2)$$

To determine the marginal density for the initial value x_1, recall that for a stationary AR (1) process:

$$E[x_1] = \mu = \frac{c}{1 - \phi}$$

$$Var(x_1) = \frac{\sigma^2}{1 - \phi^2}$$

It follows that:

$$x_1 \sim N\left(\frac{c}{1 - \phi}, \frac{\sigma^2}{1 - \phi^2}\right)$$

$$f(x_1; \theta) = \left(2\pi \frac{\sigma^2}{1 - \phi^2}\right)^{-1/2} exp\left(-\frac{1 - \phi^2}{2\sigma^2}\left(x_1 - \frac{c}{1 - \phi}\right)^2\right) \quad (3)$$

The conditional log-likelihood function is:

$$\sum_{t=2}^{T} lnf(x_t | x_{t-1}, \theta) = = \frac{-(T-1)}{2}\ln(2\pi) - \frac{(T-1)}{2} ln(\sigma^2) - \frac{1}{2\sigma^2}\sum_{t=2}^{T}(x_t - c - \phi x_{t-1})^2$$

The conditional log-likelihood function has the form of the log-likelihood function for a linear regression model with normal errors. It follows that the conditional mles for c and ϕ are identical to the least-squares estimates from the regression:

$$x_t = c + \phi x_{t-1} + \varepsilon_t, t = 2, \dots, T$$

The conditional mle for σ^2 and marginal log-likelihood for the initial value x_1 are given by equation (5) and (6) respectively.

$$\hat{\sigma}^2_{cmle} = (T-1)^{-1} \sum_{t=2}^{T} \left(x_t - \hat{c}_{cmle} - \hat{\phi}_{cmle} x_{t-1}\right)^2 \quad (5)$$

$$lnf(x_1;\theta) = -\frac{1}{2}\ln(2\pi) - \frac{1}{2} ln\left(\frac{\sigma^2}{1-\phi^2}\right) - \frac{1-\phi^2}{2\sigma^2}\left(x_1 - \frac{c}{1-\phi}\right)^2 \quad (6)$$

with exact log-likelihood function:

$$lnL(\theta|x) = -\frac{T}{2}\ln(2\pi) - \frac{1}{2} ln\left(\frac{\sigma^2}{1-\phi^2}\right) - \frac{1-\phi^2}{2\sigma^2}\left(x_1 - \frac{c}{1-\phi}\right)^2 - \frac{(T-1)}{2} ln(\sigma^2) - \frac{1}{2\sigma^2}\sum_{t=2}^{T}(x_t - c - \phi x_{t-1})^2 \quad (7)$$

The exact log-likelihood function is a non-linear function of the parameters θ, thus there is no closed-form solution for the exact mles. A Newton-Raphson type algorithm is used for the maximization which leads to the iterative scheme:

$$\hat{\theta}_{mle,n} = \hat{\theta}_{mle,n-1} - \hat{H}\left(\hat{\theta}_{mle,n-1}\right)^{-1} \hat{S}\left(\hat{\theta}_{mle,n-1}\right)$$

where $\hat{H}(\hat{\theta})$ is an estimate of the Hessian matrix (2nd derivative of the log-likelihood function), and $\hat{S}(\hat{\theta})$ is an estimate of the score vector (1st derivative of the loglikelihood function). The estimates of the Hessian and Score are computed numerically using numerical derivative routines.

Prediction error decomposition. For general time series models, the log-likelihood function is computed using an algorithm known as the prediction error decomposition. To illustrate this algorithm, consider again the simple AR (1) model. Recall,

$$x_t | I_{t-1} \sim N(c + \phi x_{t-1}, \sigma^2), t = 2, \dots, T$$

From which it follows that

$$E[x_t|I_{t-1}] = c + \phi x_{t-1}$$
$$Var[x_t|I_{t-1}] = \sigma^2$$

The 1-step ahead prediction errors may then be defined as

$$v_t = x_t - E[x_t|I_{t-1}] = x_t - c + \phi x_{t-1}, t = 2, \dots, T$$

The variance of the prediction error at time t is

$$f_t = var(v_t) = var(\varepsilon_t) = \sigma^2, t = 2, \dots . T$$

For the initial value, the first prediction error and its variance are

$$v_1 = x_1 - E[x_1] = x_1 - \frac{c}{1-\phi}$$
$$f_1 = var(v_1) = \frac{\sigma^2}{1-\phi^2}$$

Using the prediction errors and the prediction error variances, the exact log-likelihood function is re-expressed as:

$$lnL(\theta|x) = -\frac{T}{2}\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{T} lnf_t - \frac{1}{2}\sum_{t=1}^{T}\frac{v_t^2}{f_t}$$

which is the prediction error decomposition. Further simplification is achieved by:

$$var(v_t) = \sigma^2 f_t^*$$
$$= \sigma^2 . \frac{1}{1-\phi^2} \; for \; t = 1$$
$$= \sigma^2 . for \; t > 1$$

That is $f_t^* = 1/(1-\phi^2)$ for $t = 1$ and $f_t^* = 1$ for $t > 1$. Thus the log-likelihood becomes

$$lnL(\theta|x) = -\frac{T}{2}\ln(2\pi) - \frac{T}{2}ln\sigma^2 - \frac{1}{2}\sum_{t=1}^{T} ln\, f_t^* - \frac{1}{2\sigma^2}\sum_{t=1}^{T}\frac{v_t^2}{f_t^*}$$

Logistic model

The Logistic model is given by (1):

$$f(t) = \frac{\alpha}{1+\beta e^{\gamma t}} \quad 1 \le t \le T \tag{9}$$

where α is the saturation level, β and γ are parameters of the model to be estimated, t is the time in years. In the Logistic model, α is estimated by a Fibonacci search technique.

Differentiating equation (9) to t and natural logarithms are taken on both sides, we have:

$$ln\frac{df(t)}{dt} = 2lnf(t) + \delta + \gamma t \tag{10}$$

where $\delta = \ln\left(\frac{-\beta\gamma}{\alpha}\right)$

Harvey logistic model. The Harvey Logistic model is based on the Logistic model. From equation (10), the Harvey Logistic model is:

$$lny_t = 2lnY_{t-1} + \delta + \gamma t + \varepsilon_t, t = 2 \ldots\ldots T$$

where Y_t is the data to be predicted at year t, $y_t = Y_t - Y_{t-1}, t = 2 \ldots\ldots T$, ε_t is a disturbance term with zero mean and constant variance, δ and γ are constants to be found by regression.

Harvey model. The Harvey model based on generally modified exponentials is of the form:

$$f(t) = \alpha(1 + \beta e^{\gamma t})^k$$

The value of k determines the form of the function $f(t)$. When $k = -1, f(t)$ is Logistic and when $k = 1$ it is a simple modified exponential.

Differentiating and taking natural logarithm as for the Logistic model, leads to the Harvey model based on the simple modified exponential. Thus, the Harvey model is given by:

$$lny_t = \rho lnY_{t-1} + \delta + \gamma t + \varepsilon_t$$

$t = 2 \ldots\ldots T$

where $\rho = \frac{k-1}{k}, \delta = ln(k\beta\alpha^{1/k}\gamma)$

ρ, β and γ are parameters of the model to be estimated, ε_t is the error term with mean zero and constant variance.

Maximum likelihood estimation for Harvey models. Electricity generation and consumption based on the Harvey model is generally given as:

$$f(t) = \alpha(1 + \beta e^{\gamma t})^k \tag{11}$$

The proposed model is given as:

$$\ln x_t = \alpha \ln x_{t-1} + \beta + \gamma t + \varepsilon_t \tag{12}$$

where α, β and γ are the parameters of the model to be estimated. ε_t is the error term with mean zero and constant variance.

Since $x_t = X_t - X_{t-1}$, substituting for x_t in equation (12) gives:

$$\ln(X_t - X_{t-1}) = \alpha \ln(X_{t-1}) + \beta + \gamma t + \varepsilon_t \tag{13}$$

$$\ln\left(\frac{X_t}{X_{t-1}}\right) = \alpha \ln(X_{t-1}) + \beta + \gamma t + \varepsilon_t \tag{14}$$

Therefore, parameters of the Harvey model in equation (14) are estimated using the maximum likelihood method as shown below.

Taking the likelihood of equation (11),

$$L[f(t,\alpha,\beta,\gamma)] = \prod_{i=1}^{n}(\alpha(1 + \beta e^{\gamma t})^k) = \prod_{i=1}^{n}[\alpha(1 + \beta e^{\gamma t})^k] \tag{15}$$

Let $x = \beta e^{\gamma t}$

Equation (15) becomes,

$$L[f(t,\alpha,\beta,\gamma)] = \prod_{i=1}^{n}[\alpha(1 + x)^k] \tag{16}$$

Recall the binomial expansion of $(1+x)^{-n}$,

i.e. $(1+x)^{-n} = 1+\frac{-nx}{1!}+\frac{(-n(-n-1)x^2)}{2!}+\frac{(-n(-n-1)(-n-2)x^3)}{3!}+\cdots+\frac{(-n(-n-1)(-n-r)x^r)}{r!}$

However, from equation (16),

$$(1+x)^k = 1+\frac{xk}{1!}+\frac{(k(k-1)x^2)}{2!}+\frac{(k(k-1)(k-2)x^3)}{3!}+\cdots+\frac{(k(k-1)(k-2)x^r)}{r!}$$

$$\alpha(1+x)^k = \alpha+kx\alpha+\frac{\alpha(k(k-1)x^2)}{2!}+\frac{\alpha(k(k-1)(k-2)x^3)}{3!}+\cdots+\frac{\alpha(k(k-1)(k-2)x^r)}{r!}$$

$$\alpha(1+x)^k = \alpha+kx\alpha+\frac{\alpha(k(k-1)x^2)}{2}+\frac{\alpha(k(k-1)(k-2)x^3)}{6}$$

$$\prod_{1=1}^{n}\alpha(1+x)^k = \alpha^n+\prod_{i=1}^{n}\left[1+xk+\frac{k(k-1)x^2}{2}+\frac{k(k-1)(k-2)x^3}{6}\right]$$

$$\alpha^n(1+x)^{kn} = \alpha^n+\prod_{i=1}^{n}\left[1+xk+\frac{k(k-1)x^2}{2}+\frac{k(k-1)(k-2)x^3}{6}\right]$$

Still from equation (16),

$$ln[f(t,\alpha,\beta,\gamma)] = ln\alpha^n(1+x)^{kn} = ln\alpha^n+ln\left[1+xk+\frac{k(k-1)x^2}{2}+\frac{k(k-1)(k-2)x^3}{6}\right]$$

$$= nln\alpha+kn\left[x-\frac{x^2}{2}+\frac{x^3}{3}\right]$$

$$= nln\alpha+xkn-\frac{x^2kn}{2}+\frac{x^3kn}{3}$$

$$lnL[f(t,\alpha,\beta,\gamma)] = nln\alpha+\beta e^{\gamma t}kn-\frac{\beta^2e^{\gamma^2t^2}kn}{2}+\frac{\beta^3e^{\gamma^3t^3}kn}{3}$$

$$= nln\alpha+kn\left[\beta e^{\gamma t}-\frac{\beta^2e^{\gamma^2t^2}}{2}+\frac{\beta^3e^{\gamma^3t^3}}{3}\right]$$

$$\frac{lnL[f(t,\alpha,\beta,\gamma)]}{\partial(\alpha,\beta,\gamma)} = \frac{\partial}{\partial(\alpha,\beta,\gamma)}\left\{nln\alpha+kn\left[\beta e^{\gamma t}-\frac{\beta^2e^{\gamma^2t^2}}{2}+\frac{\beta^3e^{\gamma^3t^3}}{3}\right]\right\} \quad (17)$$

Therefore,

$$\frac{lnL[f(t,\alpha,\beta,\gamma)]}{\partial(\alpha)} = nln\alpha+kn\left[\beta e^{\gamma t}-\frac{\beta^2e^{\gamma^2t^2}}{2}+\frac{\beta^3e^{\gamma^3t^3}}{3}\right]=0$$

$$\frac{lnL[f(t,\alpha,\beta,\gamma)]}{\partial(\beta)} = nln\alpha+kn\left[\beta e^{\gamma t}-\frac{\beta^2e^{\gamma^2t^2}}{2}+\frac{\beta^3e^{\gamma^3t^3}}{3}\right]=0$$

$$\frac{lnL[f(t,\alpha,\beta,\gamma)]}{\partial(\gamma)} = nln\alpha+kn\left[\beta e^{\gamma t}-\frac{\beta^2e^{\gamma^2t^2}}{2}+\frac{\beta^3e^{\gamma^3t^3}}{3}\right]=0$$

Newton Raphson Iterative procedure technique for solving equations numerically is used to estimate the parameters $(\hat{\alpha},\hat{\beta},\hat{\gamma})$ of the model in equation (17).

However, estimated parameters would then reflect in equation (14) above to become:

$$\ln\left(\frac{X_t}{X_{t-1}}\right) = \hat{\alpha}ln(X_{t-1})+\hat{\beta}+\hat{\gamma}t \quad (18)$$

for the appropriate prediction of electricity generation and consumption of the Harvey model.

Markov chain. A Markov chain is a sequence of random variables $X_1, X_2, X_3, \ldots$, with the Markov property namely that, the conditional probability of any future event, given any past event and the present state, is independent of the past event and depends only on the present state. In other words, the present state is only dependent on the last state and does not depend on the states before the last state.

Let X_t denotes a random variable which represents the state of a system at time t, where $t = 0,1,2,\ldots\ldots$ If X_{t+1} only depends on the state of X_t, and does not depend on the states before X_t, then:

$$P(X_{n+1} = x|X_1 = x_1, X_2 = x_2,\ldots, X_n = x_n) = P(X_{n+1} = x_{n+1}|X_n = x_n) \quad (19)$$

X_t is a stationary Markov chain (or time-homogenous Markov chain). Let p_{ij} denotes the probability that the system is in a state j at the time $t+1$ given the system is in state i at time t. If the system has a finite number of states, $1,2,\ldots s$, the stationary Markov chain is defined by a transition probability matrix:

$$P = \begin{bmatrix} P_{11} & P_{12} & \ldots P_{1s} \\ P_{21} & P_{22} & \ldots P_{2s} \\ \vdots & \vdots & \ldots \vdots \\ P_{s1} & P_{s2} & \ldots P_{ss} \end{bmatrix}$$

where $p_{ij} \geq 0, i,j \geq 0$ and

$$\sum_{j=1}^{j=s} p_{ij} = 1$$

The transition probability matrix of a stationary Markov chain can be generated from the observations of the system state $X_0, X_1, X_2, \ldots, X_n$, at time $t = 0, \ldots, N-1$, we get the transition probability matrix as follows:

$$P_{ij} = \frac{N_{ij}}{N_i}$$

where N_{ij} is the number of observation pairs X_t and X_{t+1} with X_t in state i and X_{t+1} in state j; N_i is the number of observation pairs X_t and X_{t+1} with X_t in state i and X_{t+1} in any state.

Maximum likelihood estimation for Markov chain.

Derivation of the MLE for Markov chains. The transition matrix, p, is unknown, and we impose no restrictions on it, but rather want to estimate it from data. Given the matrix entries p_{ij} defined as:

$$p_{ij} = \Pr(X_{t+1} = j|X_t = i) \quad (20)$$

What we observe is a sample from the chain, $\mathrm{x}_1^n = x_1, x_2, \ldots., x_n$. This is a realization of the random variable X_1^n.

The probability of this realization is

$$Pr(\mathrm{X}_1^n = \mathrm{x}_1^n) = Pr(X_1 = x_1) \prod_{t=2}^{n} Pr(X_t = x_t|\mathrm{X}_1^{t-1} = \mathrm{x}_1^{t-1}) \quad (21)$$

$$= Pr(X_1 = x_1) \prod_{t=2}^{n} Pr(X_t = x_t|X_{t-1} = x_{t-1}) \quad (22)$$

Re-write in terms of the transition probabilities p_{ij} , to get the likelihood of a given transition matrix:

$$L(p) = Pr(X_1 = x_1) \prod_{t=2}^{n} p_{x_{t-1} x_t} \quad (23)$$

Define the transition counts $N_{ij} \equiv$ number of times i is followed by j in X_1^n , and re-write the likelihood in terms of:

$$L(p) = Pr(X_1 = x_1) \prod_{i=1}^{k} \prod_{j=1}^{k} p_{ij}^{n_{ij}} \quad (24)$$

taking the log results in (24)

$$L(p) = \log L(p) = log\, Pr(X_1 = x_1) + + \sum_{i,j} n_{ij} \log p_{ij} \quad (25)$$

Taking the derivative: $\frac{\partial L}{\partial p_{ij}} = \frac{n_{ij}}{p_{ij}}$

Setting equal to zero at $\hat{p}_{ij}$: $\frac{n_{ij}}{p_{ij}} = 0$

From above, the parameters cannot all change arbitrarily, because the probabilities of making transitions from a state have to add up to 1. That is, for each i, $\sum_j p_{ij} = 1$.

Thus, by explicitly eliminating parameters, we arbitrarily pick one of the transition probabilities to express in terms of the others; such that the probability of going to 1, we have for each i, $p_{i1} = 1 - \sum_{j=2}^{m} p_{ij}$.

Taking the derivatives of the likelihood, we leave out $\partial/\partial P_{i1}$, and the other terms will be changed:

$$\frac{\partial L}{\partial p_{ij}} = \frac{n_{ij}}{p_{ij}} - \frac{n_{i1}}{p_{i1}} \quad (26)$$

Setting this equal to zero at the MLE $\hat{p}$,

$$\frac{n_{ij}}{\hat{p}_{ij}} = \frac{n_{i1}}{\hat{p}_{i1}} \quad (27)$$

$$\frac{n_{ij}}{n_{i1}} = \frac{\hat{p}_{ij}}{\hat{p}_{i1}} \quad (28)$$

Since this holds for all $j \neq 1$, we can conclude that $\hat{p}_{ij} \propto n_{ij}$, and in fact:

$$\hat{p}_{ij} = \frac{n_{ij}}{\sum_{j=1}^{m} n_{ij}} \quad (29)$$

The choice of $\hat{p}_{i1}$ as the transition probability to eliminate in favor of the others is arbitrary and we get the same result for any other.

Performance evaluation of the model. This section presents statistical tools such as the coefficient of determination (r^2), Root Mean Square Error (RMSE), and Akaike Information Criteria (AIC) to evaluate the models discussed in the previous section.

Coefficient of determination. The coefficient of determination (r^2) is used to determine the effectiveness of using the model in forecasting. It is the proportion of the variance in the dependent variable that is predictable from the independent variable(s). It gives the coefficient of the total variance in the dependent variable explained by the model.

$$r^2 = \frac{\sum_{t=1}^{n}(\hat{X}_t - \bar{X})^2}{\sum_{t=1}^{n}(X_t - \bar{X})^2} \quad (30)$$

where, $\hat{X}_t$ and X_t are the estimated and actual value of generation or consumption of electricity data, while n is the number of observations or data points. The higher the value of the coefficient of determination, the better the model.

Root mean square error (RMSE). The Root Mean Square Error (RMSE) of an estimator measures the average of the squares of the errors or deviations. That is the difference between the estimator and what is estimated. If $\hat{X}$ is a vector of n predictions, and X is the vector of observed values corresponding to the inputs to the function which generated the predictions, then the RMSE of the predictor is estimated by:

$$\frac{\sum_{i=1}^{n}(X_t - \hat{X}_t)^2}{n} \quad (31)$$

where, $\hat{X}_t$ and X_t are the estimated and actual value of generation or consumption of electricity data, while n is the number of observations or data points. The lower the value of the root mean square error, the better the model.

Akaike information criteria (AIC). The Akaike Information Criterion (AIC) is a measure of the relative quality of statistical models for a given set of data. Given a collection of models for the data, AIC estimates the quality of each model, relative to each of the other models.

Suppose that we have a statistical model M of some data x, let k be the number of estimated parameters in the model. Let $\hat{L}$ be the maximized value of the likelihood function for the model; i.e. $\hat{L} = P(x/\hat{\theta}, M)$ where $\hat{\theta}$ are the parameter values that maximize the likelihood function. Then, the AIC value of the model results in (32):

$$AIC = 2k - 2\ln(\hat{L}). \quad (32)$$

RESULTS AND DISCUSSION

The proposed models (Harvey model, Autoregressive model, and Markov chain model) discussed in the previous section are applied to model electricity generation and consumption in Nigeria between 1990 and 2017. The data was extracted from the archives of the Central Bank of Nigeria, and the National Bureau of Statistics. The volume of electricity generated and consumed between 1990 and 2017 constitutes the historical data set. The data set is used to compare the prediction accuracy of the three models. The models would be fitted on the historical data of electricity generation and consumption in Nigeria and the best model would be used to forecast electricity generation and consumption for the next twenty years; (2018-2037). Figures 1 and 2 show the Electricity Generation and Consumption in Nigeria. Table 1 reported the descriptive statistics of annual electricity generation and consumption.

Table 1 – Distributional characteristics of annual electricity generation and consumption, mln kWh

Characteristics	Electricity Generation	Electricity Consumption
Mean	1,616.27	2,133.16
Standard Error	133.92	121.30
Median	1,469.39	2,064.65
Standard Deviation	708.61	641.87
Sample Variance	502,134.61	411,992.44
Kurtosis	-1.40	-1.12
Skewness	0.42	0.49
Range	2,020.39	2,014.22
Minimum	829.32	1,346.3
Maximum	2,849.72	3,360.52
Sum	45,255.59	59,728.55
Count	28	28
Confidence Level (95.0%)	274.77	248.89

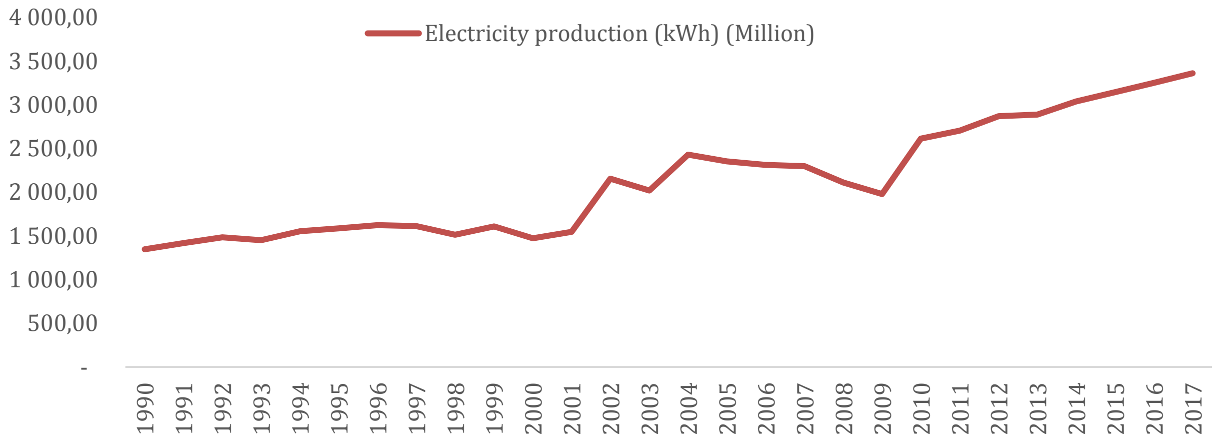

Figure 1 – Electricity Generation in Nigeria between 1990-2017

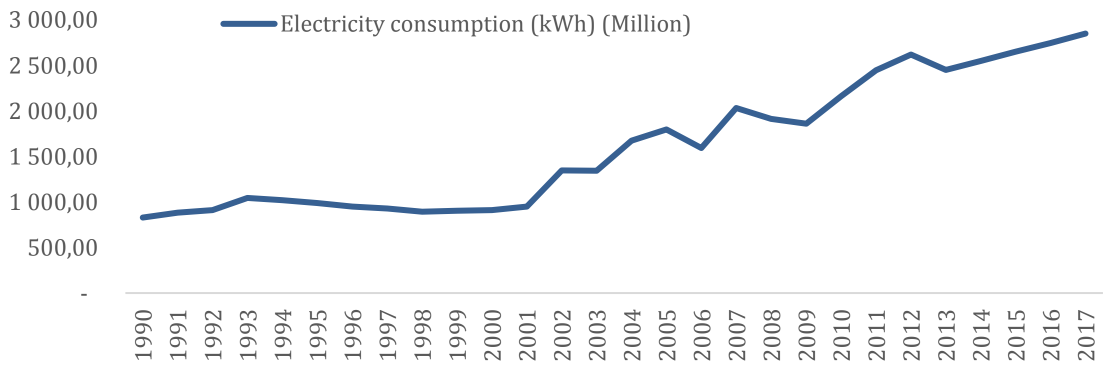

Figure 2 – Electricity Consumption in Nigeria between 1990-2017

Autoregressive model: Electricity generation

Based on the model parameters shown in Table 2, the Autoregressive model for electricity production is:

$$\widehat{X_t} = 14.5003 + 0.9403X_{t-1}$$

where $\widehat{X_t}$ is the estimated electricity generation.

Table 2 – Results for Electricity Generation Estimation using Autoregressive model

Parameter	Coefficient	Standard Error	AIC	r^2	RMSE	MAE
β0	14.5003	7.6265	157.99	0.8207162	1266.88	195.6264
β1	0.9403	0.0627				

The model gave r^2 of 0.8207 which means that the Autoregressive model was able to explain 82.1% of the variance in electricity generation. The coefficient of $\widehat{X_t}$, 0.9403, reveals that the electricity generation in Nigeria increases with time. Table 3 and Figure 3 presents the value of the actual and estimated electricity generation.

Table 3 – Actual and Predicted Electricity Generation Using Autoregressive model, mln kWh

S/n	Year	Actual	Predicted	S/n	Year	Actual	Predicted
1	1990	1346.3	1219.44	15	2004	2427.5	1912.31
2	1991	1416.7	1280.43	16	2005	2353.9	2297.08
3	1992	1483.4	1346.62	17	2006	2311	2227.87
4	1993	1450.5	1409.34	18	2007	2297.8	2187.53
5	1994	1553.1	1378.41	19	2008	2111	2175.12
6	1995	1585.7	1474.88	20	2009	1977.7	1999.47
7	1996	1624.3	1505.53	21	2010	2612.1	1874.13
8	1997	1611.7	1541.83	22	2011	2703.4	2470.66
9	1998	1511.1	1529.98	23	2012	2870.6	2556.51
10	1999	1608.9	1435.39	24	2013	2888.3	2713.73
11	2000	1472.7	1527.35	25	2014	3039	2730.37
12	2001	1546.3	1399.28	26	2015	3142.6	2872.07
13	2002	2154.4	1468.49	27	2016	3249.73	2969.49
14	2003	2018.3	2040.28	28	2017	3360.52	3070.22

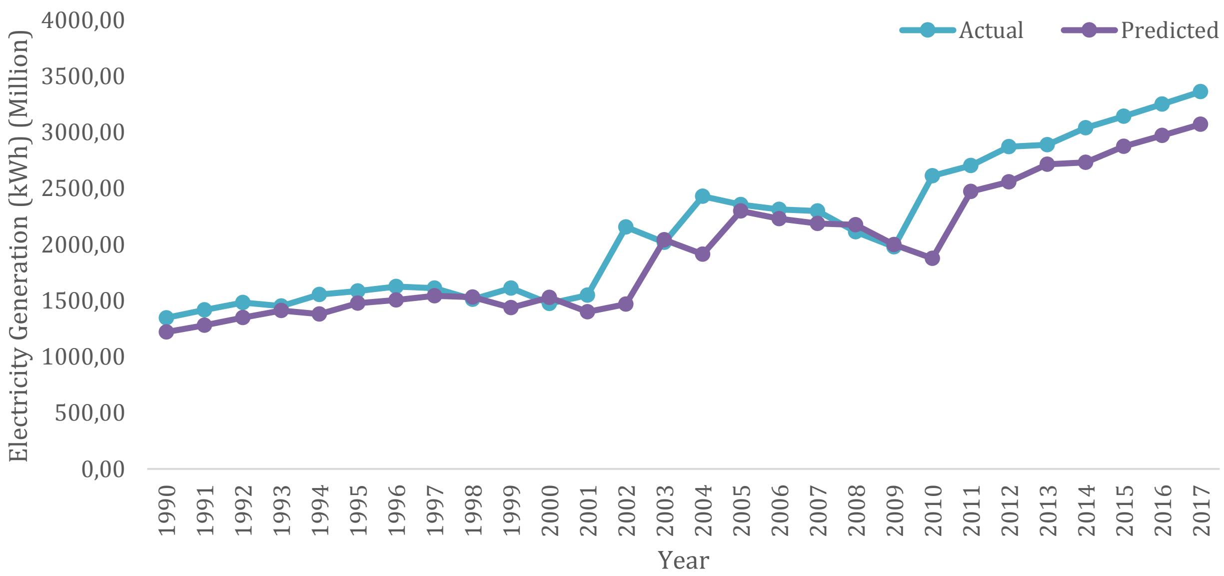

Figure 3 – Actual and Predicted Electricity Generation in Nigeria between 1990-2017

Autoregressive model: Electricity consumption

From Table 4 the Autoregressive model parameters for electricity consumption is:

$$\widehat{X_t} = 15.0727 + 0.9492X_{t-1}$$

where, $\widehat{X_t}$ is the estimated electricity consumption.

Table 4 – Results for Electricity Consumption Estimation using Autoregressive model

Parameter	Coefficient	Standard Error	AIC	r²	RMSE	MAE
β0	15.0727	7.7210	152.33	0.8494	1294.13	210.94
β1	0.9492	0.0543				

The model gave r^2 of 0.8494 which means that the Autoregressive model explains 84.9% of the variance in electricity consumption. The coefficient of $\widehat{X_t}$ which is 0.9492, implies that electricity consumption in Nigeria increases with time. The value of the actual and estimated electricity consumption is shown in Table 5 and Figure 4.

Table 5 – Actual and Predicted Electricity Consumption Using Autoregressive model, mln kWh

S/n	Year	Actual	Predicted	S/n	Year	Actual	Predicted
1	1990	829.32	903.7	15	2004	1672.55	1290.978
2	1991	884.02	802.264	16	2005	1796.03	1602.657
3	1992	910.81	854.1852	17	2006	1592.28	1719.864
4	1993	1,045.81	879.6113	18	2007	2033.55	1526.465
5	1994	1,020.39	1007.756	19	2008	1912.57	1945.318
6	1995	987.89	983.6238	20	2009	1861.02	1830.484
7	1996	950.22	952.7789	21	2010	2162.82	1781.553
8	1997	929.95	917.0173	22	2011	2446.58	2068.021
9	1998	894.57	897.7821	23	2012	2620.86	2337.366
10	1999	904.20	864.1997	24	2013	2452.17	2502.793
11	2000	911.60	873.341	25	2014	2549.72	2342.672
12	2001	947.88	880.3647	26	2015	2652.35	2435.267
13	2002	1346.50	914.8022	27	2016	2746.02	2532.683
14	2003	1344.19	1293.171	28	2017	2849.72	2621.595

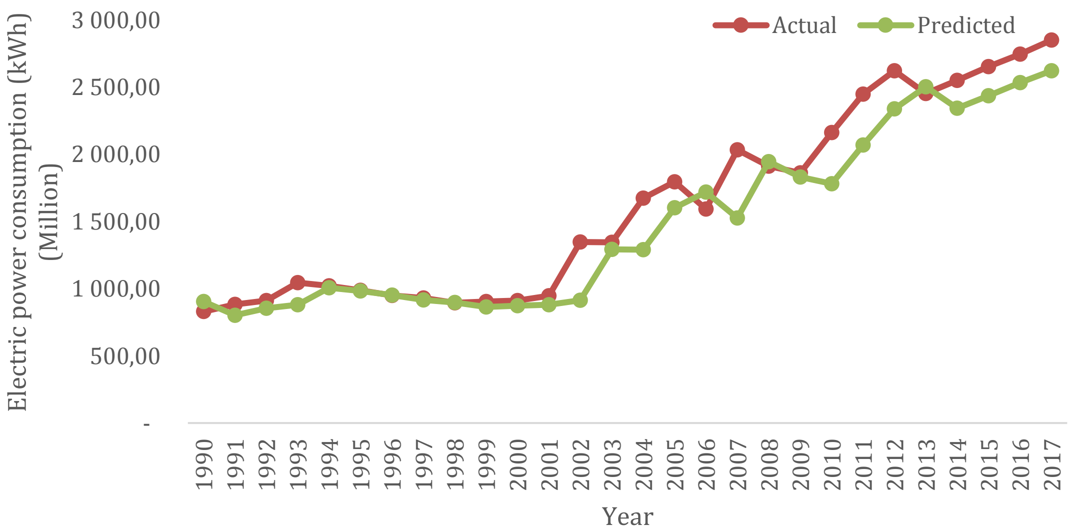

Figure 4 – Actual and Predicted Electricity Consumption in Nigeria between 1990-2017

Harvey model: Electricity production

From Table 6, with r^2 value of 0.9072, the Harvey model accounted for 90.7% of the variation in electricity generation. Moreover, the coefficient of t is positive ($\hat{\gamma}$ = 0.0155) which means that electricity generation increases with time.

Table 6 – Results for Electricity Generation Estimation using Harvey Model

Parameter	Coefficient	Standard Error	AIC	r²	RMSE	MAE
$\hat{\alpha}$	0.019984	0.03810	149.18	0.9072	1121.41	152.70
$\hat{\beta}$	-0.07355	22.6370				
$\hat{\gamma}$	0.01546	0.00563				

The Harvey model is:

$$\ln\left(\frac{X_t}{X_{t-1}}\right) = \hat{\alpha} ln(X_{t-1}) + \hat{\beta} + \hat{\gamma} t$$

$$\ln\left(\frac{X_t}{X_{t-1}}\right) = 0.0199 ln(X_{t-1}) - 0.0736 + 0.0155t$$

$$\ln\left(\frac{X_t}{X_{t-1}}\right) = 0.0199 ln(X_{t-1}) + 0.0155t - 0.0736$$

$$X_t = X_{t-1}\exp\left(\hat{\alpha} ln(X_{t-1}) + \hat{\beta} + \hat{\gamma} t\right.$$

$$X_t = X_{t-1}\, e^{(0.0199 ln(X_{t-1}) + 0.0155t - 0.07356}$$

Table 7 and Figure 5 present the actual and predicted value of electricity generation based on the Harvey model.

Table 7 – Actual and Predicted Electricity Generation Using Harvey model, mln kWh

S/n	Year	Actual	Predicted	S/n	Year	Actual	Predicted
1	1990	1346.3	1389.06	15	2004	2427.5	2217.19
2	1991	1416.7	1467.06	16	2005	2353.9	2676.57
3	1992	1483.4	1545.34	17	2006	2311	2593.83
4	1993	1450.5	1619.59	18	2007	2297.8	2545.62
5	1994	1553.1	1582.96	19	2008	2111	2530.79
6	1995	1585.7	1697.24	20	2009	1977.7	2321.11
7	1996	1624.3	1733.58	21	2010	2612.1	2171.71
8	1997	1611.7	1776.64	22	2011	2703.4	2884.34
9	1998	1511.1	1762.58	23	2012	2870.6	2987.21
10	1999	1608.9	1650.44	24	2013	2888.3	3175.77
11	2000	1472.7	1759.46	25	2014	3039	3195.74
12	2001	1546.3	1607.67	26	2015	3142.6	3365.90
13	2002	2154.4	1689.66	27	2016	3249.73	3482.97
14	2003	2018.3	2369.79	28	2017	3360.52	3604.13

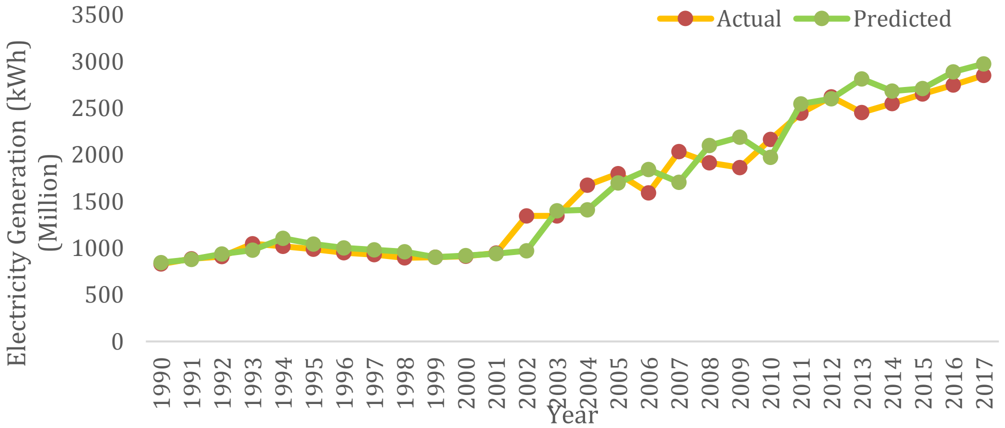

Figure 5 – Actual and Predicted Electricity Generation in Nigeria between 1990-2017

Harvey model: Electricity consumption

From Table 8, with r^2 value of 0.9485, the Harvey model accounted for 94.85% of the variation in electricity consumption. Moreover, the coefficient of t is positive ($\hat{\gamma} = 0.1979$) which means that electricity consumption increases with time.

Table 8 – Results for Electricity Consumption Estimation using Harvey Model

Parameter	Coefficient	Standard Error	AIC	r²	RMSE	MAE
$\hat{\alpha}$	0.001558	7.7210	141.22	0.9485	835.63	113.06
$\hat{\beta}$	−0.164895	24.1191				
$\hat{\gamma}$	0.197852	0.2730				

The Harvey model is:

$$\ln\left(\frac{X_t}{X_{t-1}}\right) = \hat{\alpha}ln(X_{t-1}) + \hat{\beta} + \hat{\gamma}t$$

$$\ln\left(\frac{X_t}{X_{t-1}}\right) = 0.0016ln(X_{t-1}) - 0.1649 + 0.1979t$$

$$\ln\left(\frac{X_t}{X_{t-1}}\right) = 0.0016ln(X_{t-1}) + 0.1979t - 0.1649$$

$$X_t = X_{t-1}\exp\left(\hat{\alpha}ln(X_{t-1}) + \hat{\beta} + \hat{\gamma}t\right)$$

$$X_t = X_{t-1}\, e^{(0.0016ln(X_{t-1})+0.1979t - 0.1649}$$

Table 9 and Figure 6 present the actual and predicted value of electricity consumption based on the Harvey model.

Table 9 – Actual and Predicted Electricity Consumption Using the Harvey model, mln kWh

S/n	Year	Actual	Predicted	S/n	Year	Actual	Predicted
1	1990	829.32	852.88	15	2004	1672.55	1404.91
2	1991	884.02	866.13	16	2005	1796.03	1748.70
3	1992	910.81	923.34	17	2006	1592.28	1878.01
4	1993	1045.81	951.37	18	2007	2033.55	1664.64
5	1994	1020.39	1092.62	19	2008	1912.57	2126.78
6	1995	987.89	1066.02	20	2009	1861.02	2000.06
7	1996	950.22	1032.02	21	2010	2162.82	1946.07
8	1997	929.95	992.61	22	2011	2446.58	2262.19
9	1998	894.57	971.40	23	2012	2620.86	2559.48
10	1999	904.20	934.38	24	2013	2452.17	2742.10
11	2000	911.60	944.46	25	2014	2549.72	2565.34
12	2001	947.88	952.20	26	2015	2652.35	2667.55
13	2002	1346.50	990.16	27	2016	2746.02	2775.10
14	2003	1344.19	1407.33	28	2017	2849.72	2873.26

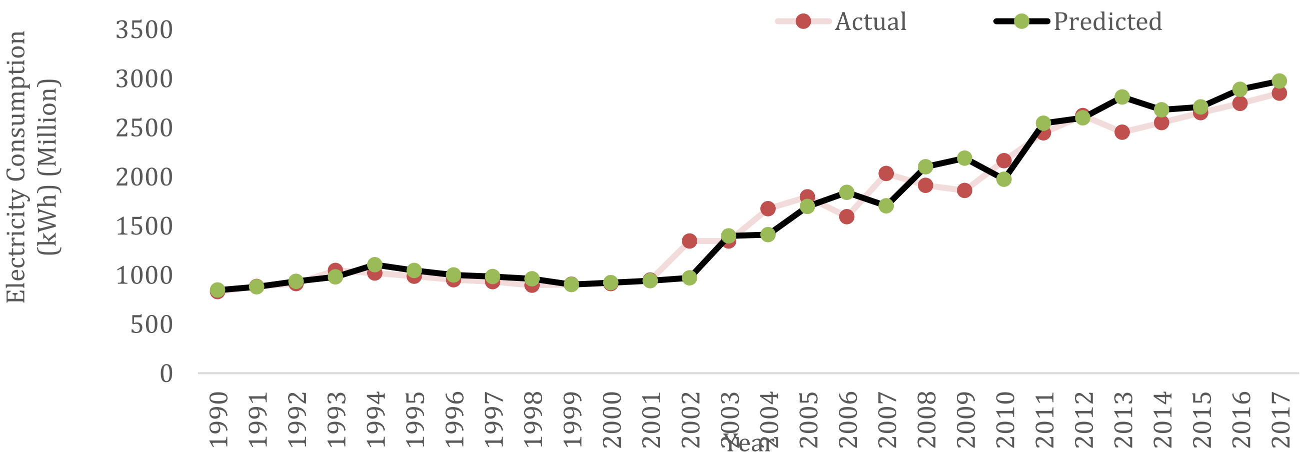

Figure 6 – Actual and Predicted Electricity Consumption in Nigeria between 1990-2017

Markov chain model

The Markov chain model is used in the prediction of generation and consumption of electricity in Nigeria. Based on the average generation and consumption of electricity, states were classified into five possible states (1, 2, 3, 4, 5). The generation volumes are expressed as 1 = very low (≤1,500 MWh), 2 = low (1,501-2,000 MWh), 3 = middle (2,001-2,500 MWh), 4 = high (2,501-3,000 MWh), 5 = very high (≥3,000 MWh). Similarly, the consumption volume of electricity were classified into five possible states (1, 2, 3, 4, 5), and expressed as 1 = very low (≤1,000 MWh), 2 = low (1,001-1,500 MWh), 3=middle (1,501-2,000 MWh), 4 = high (2,001-2,500 MWh), 5 = very high (≥2,500 MWh). If X_t denotes the state of the volume of electricity generated and consumed for a given year, X_t is a random variable describing the electricity generated and consumed on the t^{th} period and is termed as "the state" of the process.

Electricity Generation. Below is the transition matrix of electricity generation defined by the following states: Very Low, Low, Middle, High, Very High

The transition matrix defined as follows:

	Very Low	Low	Middle	High	Very High
Very Low	0.5000	0.20	0.3000	0	0
Low	0.4625	0.24	0.2975	0	0
Middle	0.4375	0.20	0.3625	0	0
High	0.4375	0.20	0.3625	0	0
Very High	0.5000	0.19	0.3100	0	0

Table 10 – Results for Electricity Generation Estimation using Markov Chain Model

Parameter	Coefficient	Standard Error	AIC	r²	RMSE	MAE
1300-1500 (Very low)	0.8	0.4010	142.1	0.9613	1027.32	193.44
1501-2000 (Low)	1.075	0.5307				
2001-2500 (Middle)	0.9821	0.4749				
2501-3000 (High)	0.8929	0.5758				
3000+ (Very High)	1.2500	0.8273				

From Table 10, with r^2 value of 0.9613, it means that the Markov chain model accounted for 96.13% of the variance in electricity generation. The coefficient of the parameter at the space-time is positive (1.2500) which means that electricity generation in Nigeria increases with the same space-time. The value of the actual and estimated electricity generation are shown in Table 11 and Figure 7.

Table 11 – Actual and Predicted Electricity Generation Using Markov Chain Model, mln kWh

S/n	Year	Actual	Predicted	S/n	Year	Actual	Predicted
1	1990	1346.3	1415.23	15	2004	2427.5	2476.11
2	1991	1416.7	1487.46	16	2005	2353.9	2323.16
3	1992	1483.4	1522.10	17	2006	2311	2611.47
4	1993	1450.5	1593	18	2007	2297.8	2418.30
5	1994	1553.1	1630.45	19	2008	2111	2599.20
6	1995	1585.7	1622.80	20	2009	1977.7	2227.38
7	1996	1624.3	1693.07	21	2010	2612.1	2010.49
8	1997	1611.7	1736.91	22	2011	2703.4	2882.11
9	1998	1511.1	1710.3	23	2012	2870.6	2934
10	1999	1608.9	1662.13	24	2013	2888.3	3129.68
11	2000	1472.7	1777.9	25	2014	3039	3481.71
12	2001	1546.3	1563.82	26	2015	3142.6	3421.84
13	2002	2154.4	1700.74	27	2016	3249.73	3305.12
14	2003	2018.3	2311.19	28	2017	3360.52	3725.09

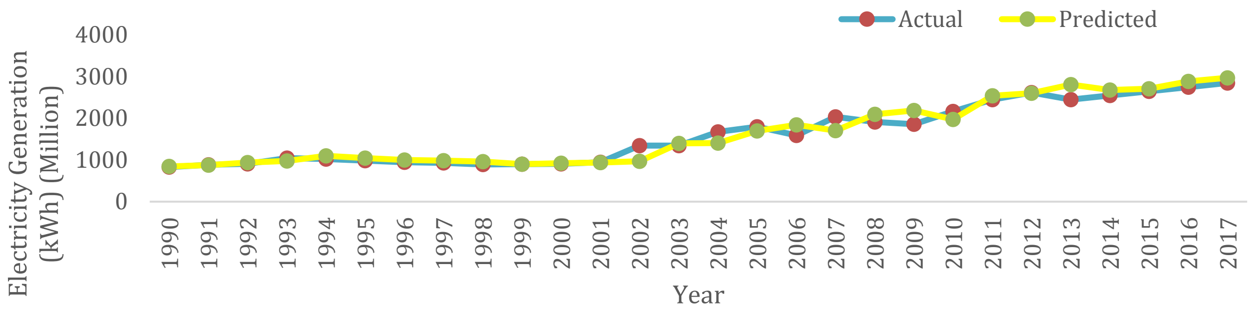

Figure 7 – Actual and Predicted Electricity Generation in Nigeria between 1990-2017

Electricity Consumption. Below is the transition matrix of electricity consumption defined by the following states: Very Low, Low, Middle, High, and Very High.

From Table 12, with r^2 value of 0.9417, implies that the Markov chain model accounted for 94.17% of the variance in electricity consumption. Moreover, the coefficient of the parameter at the space-time is positive (1.2500) which means that electricity consumption in Nigeria increases with the same space-time.

The transition matrix defined as follows:

	Very Low	Low	Middle	High	Very High
Very Low	0.375	0.20	0.3375	0.0875	0
Low	0.275	0.24	0.2925	0.1925	0
Middle	0.375	0.20	0.3375	0.0875	0
High	0.375	0.20	0.3375	0.0875	0
Very High	0.400	0.19	0.2800	0.1300	0

The value of the actual and estimated electricity consumption using the Markov chain is shown in Table 13 and Figure 8.

Table 12 – Electricity Consumption Estimation using Markov Chain Model

Parameter	Coefficient	Standard Error	AIC	r^2	RMSE	MAE
800-1000 (Very low)	0.9	0.3010	150.42	0.9417	888.89	123.05
1001-1500 (Low)	0.85	0.5330				
1501-2000 (Middle)	1.05	0.6501				
2001-2500 (High)	0.95	0.6330				
2501+ (Very High)	1.250	0.7500				

Table 13 – Actual and Predicted Electricity Consumption Using Markov Chain Model, mln kWh

S/n	Year	Actual	Predicted	S/n	Year	Actual	Predicted
1	1990	829.32	845.10	15	2004	1672.55	1410
2	1991	884.02	877.89	16	2005	1796.03	1698.57
3	1992	910.81	935.28	17	2006	1592.28	1840.27
4	1993	1045.81	979.23	18	2007	2033.55	1704.98
5	1994	1020.39	1103.5	19	2008	1912.57	2100.50
6	1995	987.89	1043.83	20	2009	1861.02	2189.10
7	1996	950.22	1001.34	21	2010	2162.82	1973.56
8	1997	929.95	982.24	22	2011	2446.58	2544.80
9	1998	894.57	960.18	23	2012	2620.86	2599.30
10	1999	904.20	901.39	24	2013	2452.17	2811.11
11	2000	911.60	920.67	25	2014	2549.72	2680.12
12	2001	947.88	939.56	26	2015	2652.35	2709.76
13	2002	1346.50	971.88	27	2016	2746.02	2888.71
14	2003	1344.19	1398.45	28	2017	2849.72	2973.87

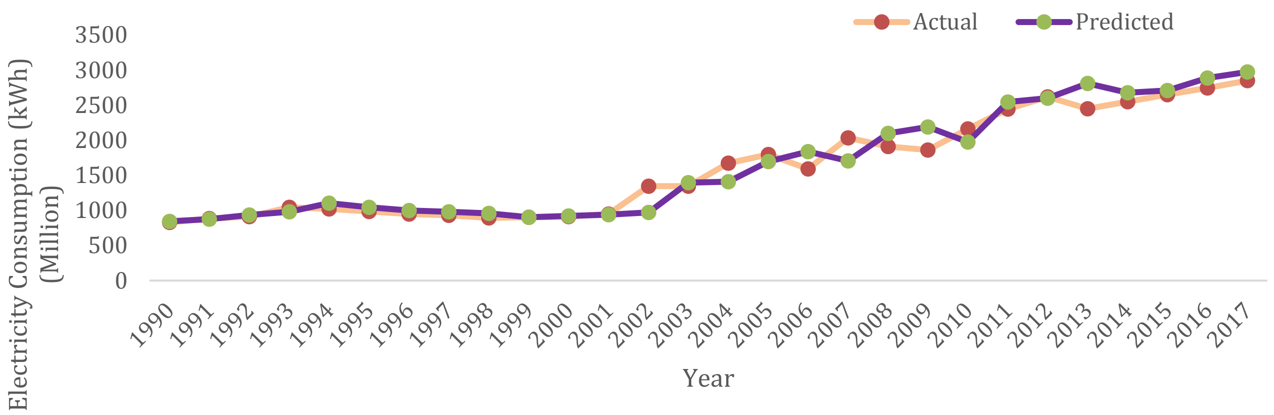

Figure 8 – Actual and Predicted Electricity Consumption in Nigeria between1999-2017

Tables 14 and 16 compared the appropriateness of Autoregressive, Harvey, and Markov models on electricity generation and consumption in Nigeria. Specification measures such as; Coefficient of determination (r^2), Root Mean Square Error (RMSE), and Akaike Information Criterion (AIC) were applied.

Table 14 reveals that the Markov chain model predicted better than the Harvey and Autoregressive models for electricity generation, as it gave a higher value of the coefficient of determination (r^2=96.0%), lower Root Mean Square Error (1027.32), and Akaike Information Criterion (142.1).

The forecasting of electricity generation is obtained from the Markov chain model by extrapolating the data from the year 2018 to 2037. Table 15 shows the forecast for electricity generation in Nigeria using the best-selected Model (Markov Chain Model). The forecast values in Table 15 and Figure 9 indicate that electricity generation in Nigeria is continuously increasing. Electricity generation in Nigeria will increase from 3,692.11 mln kW/h in 2018 to 18,250.67 mln kW/h in 2037.

Table 14: Comparison of the Forecasting Accuracy of the Autoregressive model, Harvey and Markov Chain Model for Electricity Generation

Model	AIC	r^2	RMSE
Autoregressive	157.99	0.82	1,266.88
Harvey	149.18	0.91	1,121.41
Markov Chain	142.1	0.96	1,027.32

Table 15: Forecast of Electricity production using the Markov chain Models (2018-2037)

S/n	Year	Forecast of Electricity Generation, mln kWh	S/n	Year	Forecast of Electricity Generation, mln kWh
1	2018	3,692.11	11	2028	7,037.88
2	2019	3,411.37	12	2029	8,293.03
3	2020	3,210.68	13	2030	9,630.90
4	2021	3,002.87	14	2031	10,793.30
5	2022	3,000.43	15	2032	11,820.40
6	2023	3,220.20	16	2033	12,950.46
7	2024	3,792.00	17	2034	14,987.74
8	2025	4,832.56	18	2035	15,503.56
9	2026	5,503.00	19	2036	16,689.19
10	2027	5,997.19	20	2037	18,250.67

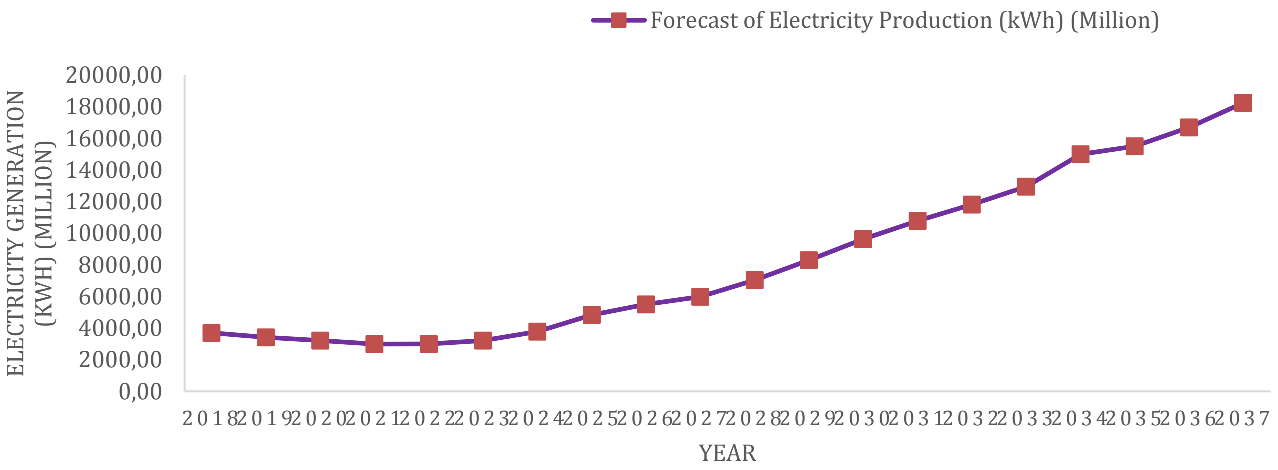

Figure 9 – The plot of Forecasted Electricity Generation in Nigeria from 2018 – 2037

Table 16 reveals that the Harvey model predicted better than the Markov and Autoregressive models for electricity consumption, as it gave a higher value of the coefficient of determination (r^2=95.0%), lower Root Mean Square Error (835.63), and Akaike Information Criterion (141.22).

Table 17 shows the forecast of electricity consumption in Nigeria using the best-selected Model (Harvey model). The forecasting of electricity consumption is obtained from the Harvey model by extrapolating the data from the year 2018 to 2037. The forecast values in Table 17 and Figure 10 indicate that electricity consumption in Nigeria is continuously increasing. Electricity consumption in Nigeria will increase from 2,961.10 mln kW/h in 2018 to 127,071.30 mln kW/h in 2037.

Table 16 – Comparison of the Forecasting Accuracy of the Autoregressive model, Harvey and Markov Chain Model for Consumption

Model	AIC	r^2	RMSE
Autoregressive	152.33	0.85	1294.13
Harvey	141.22	0.95	835.63
Markov Chain	148.42	0.94	888.89

Table 17 – Forecast of Electricity consumption using the Harvey Models (2018-2037)

S/n	Year	Forecast of Electricity Consumption, mln kWh	S/n	Year	Forecast of Electricity Consumption, mln kWh
1	2018	2,961.10	11	2028	21,414.76
2	2019	3,608.93	12	2029	26,099.92
3	2020	4,398.50	13	2030	31,810.12
4	2021	5,360.82	14	2031	38,769.60
5	2022	6,533.67	15	2032	47,251.69
6	2023	7,963.12	16	2033	57,589.50
7	2024	9,705.30	17	2034	70,189.05
8	2025	11,828.65	18	2035	85,545.15
9	2026	14,416.54	19	2036	104,260.89
10	2027	17,570.62	20	2037	127,071.30

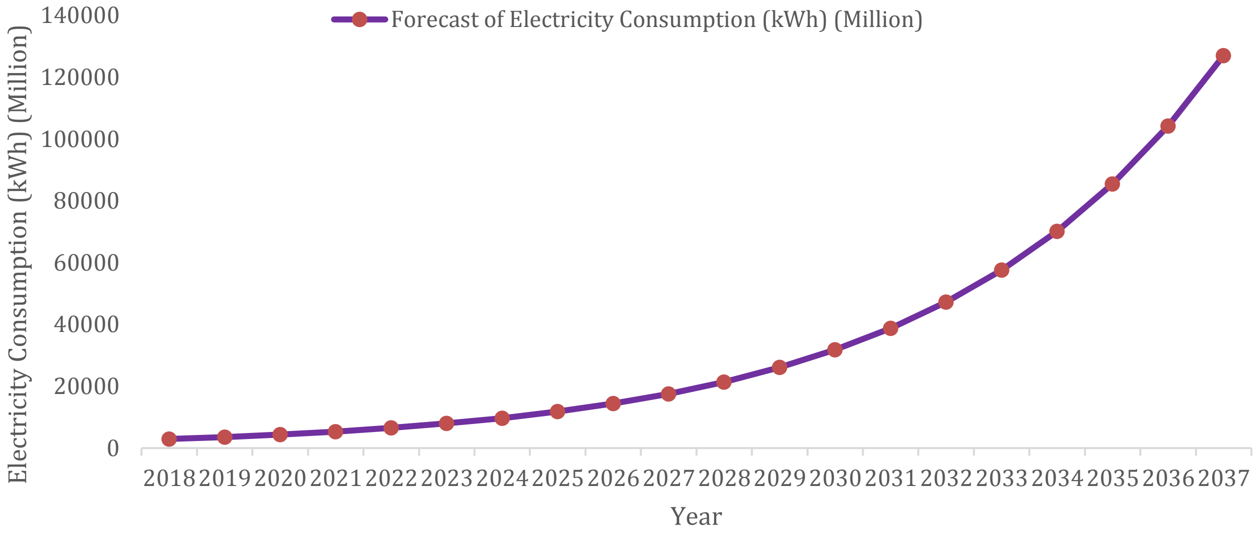

Figure 10 – The plot of Forecasted Electricity Consumption in Nigeria from 2018–2037

CONCLUSION

Forecasting electricity generation and consumption is an important component of the electricity market, as it helps to plan production along with required demand and to prevent energy wastage and system failure. This paper has investigated the effectiveness and validation of three different models; Markov chain, Harvey, and Autoregressive models in modeling electricity generation and consumption in Nigeria. From the analysis performed, it was discovered that the coefficient of t is positive which means that electricity generation and consumption increases with time. There is strong evidence in favor of the fact that there is an increase in demand and consumption of electricity in Nigeria. Again, modeling historical data on generation and consumption was better explained by the Markov chain model for the generation data and Harvey model for the consumption data. This corresponds to what can be observed from the time series plot in Figures 9 & 10 respectively, which shows the trend in generation and consumption of electricity. The Markov chain and Harvey models also performed better in the prediction of electricity generation and consumption. Hence, the two models are better for describing generation and consumption volume of electricity respectively.

Based on the results obtained from Markov and Harvey models in predicting generation and consumption of electricity, we can conclude that the demand for electricity in Nigeria will maintain an upswing over time. This is evident in the historical data which shows that generation and consumption have majorly been on the increase yearly.

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