

The Stability of Columns Against Lateral Impacts and Vertical Loads of a Three-Storey Building Using Euler and Vianello Method

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Abstract. Attacks on city centres using vehicle bombs have become characteristic of the campaigns of various organisations of international terrorism. An explosion of a bomb inside or directly near a structure may lead to disastrous consequences for the external and internal structural frames of the building, as well as the collapse of walls, the explosion of large panels of windows, and the disconnection of key life-safety systems. Numerous factors, including direct blast effects, collapse of the structure, impacts of the debris, fire, and smoke, may cause death and physical injuries to the occupants.

The indirect impacts may merge to block or hinder people from evacuating on time, thus leading to additional losses. Moreover, significant disasters caused by the explosion of gases + chemicals lead to high dynamic loads, exceeding the initial design loads, of most constructions. Such extreme loading conditions pose significant danger, and hence, over the last 30 years, efforts have been underway to study ways of structural analysis and design to resist blast loads. Blast analysis and design of buildings subjected to blast impact loading demand an extensive knowledge of blast phenomena and the dynamic response of several structural components. The paper gives an informative account of the impact of an explosion on buildings. The nature of explosions and how the blast waves in free air work is explained. This paper also presents diverse methods for estimating blast loads and structural response.

Keywords:

INTRODUCTION

The last few years have seen increasing interest worldwide in the stability of buildings subject to blast and impact loads, especially as the threat of terrorism has continued to grow. Mobile, highly potent, and miniature explosive devices have been used more, and they increase the vulnerability of the civilian infrastructure to devastating effects. This issue is particularly acute in those areas, such as Adamawa, where these threats have become real with catastrophic attacks. Such explosions are not only fatal due to the sudden energy discharge but also far more crucial because they result in structural failures that compound the level of damage. The mass destruction levels that resulted from the World Trade Centre bombing of 1993 and the Oklahoma City bombing of 1995 depict the serious nature of structural system instability when the lateral impact

loads tamper with the integrity of crucial elements such as columns.

Explosions quickly discharge energy and are generally classified as physical, chemical, or nuclear. Among these, chemical explosions are the most applicable in civil engineering, occurring as a result of the combustion of fuel components such as carbon and hydrogen. High explosives are those that produce extreme temperatures and pressures during detonation, resulting in a concentrated shock wave, such as TNT and ANFO. The overpressure originating from this wave first causes an immediate, extensive surge in pressure, followed by a period of negative pressure, culminating in intense suction forces that can even carry debris to great distances. The effect of alternate pressure caused by such explosions can cause buildings to collapse, thereby undermining the understanding and design of countermeasures to mitigate their action.

One of the key structural failures during these loading types is buckling, in which compression elements, such as columns, deflect sideways in response to an axial load or lateral load. A geometry instability known as buckling restricts the ability of the structure to sustain extra load. It may take place under elastic conditions, that is, below the proportional limit of the material, or under so-called inelastic conditions when irreversible deformations occur. Researchers commonly study elastic buckling in idealised extended, slender members, but real columns with shorter lengths or residual stresses more often experience inelastic buckling. Factors such as initial curvature, loading eccentricity, and variations in material properties further increase the chances of buckling. The two most frequently applied analytical methods for measuring buckling behaviour are the Euler method and the Vianello method. The classical formula, due to Euler, is the load at which a perfectly straight, pin-ended, prismatic column with infinitely thin walls starts buckling elastically. It provides an easy-to-use yet practical estimation under idealised conditions and small deformations. This method, however, is too optimistic because such things are difficult to find in real-life situations. The method proposed by Vianello, however, is more fine-tuned and iterative, making it more suitable for members with non-uniform stiffness or those with complex loading and support conditions. Despite being more complicated and less familiar, this approach is more accurate where assumptions by Euler do not hold.

The structural stability concept is applied directly in this case. The stability of a structure is denoted by its ability to return to equilibrium after being disturbed. Several structures, depending on their geometry and the load, may be in stable/unstable/neutral equilibrium. Tiny oscillations in an unstable system can result in large, irreversible deformations, which underscores the importance of designing for resilience in the face of unexpected or even extreme loading conditions. Relatively long compression members are most susceptible to the buckling effect because they are slender, and a combination of geometric imperfection and imperfect loads can be cited as the causes behind a majority of accidents.

Engineers can enhance structural efficiency by using tapered columns and deploying members with varying cross-sections to concentrate material where it is most needed, thereby counteracting bending and buckling. This solution not only

averts poor performance, but also minimises materials to drive more eco-efficient designs; this is particularly essential in high-performance buildings such as towers and aircraft, where the weight is vital.

The present study aims to investigate the stability of columns in a three-storey building subjected to combined vertical (axial) and lateral (blast-induced) loads, utilising both analytical methods proposed by Euler and Vianello. The objective is to compare their accuracy and effectiveness in predicting the critical buckling load under such complex situations. Specifically, the study aims to understand how lateral forces that occur at the time of the blast influence the situation, estimate the vertical loads at the moment of detonation, and determine the acceptable buckling deformation resulting from the interaction of these loads. The results will provide engineers and architects with more informed insights into structural design strategies that can help them achieve safety and resilience in blast-prone areas.

Such analysis is justified by the fact that there is an ever-growing need for safe and economical structural systems that do not collapse due to sudden and extreme loading. In the event of an explosive threat, the integrity of a column in any structure may well determine whether a building will collapse immediately or remain standing long enough to facilitate the safe evacuation of victims. Thus, it can be said that an in-depth study of column behaviour due to combined loading is both technically significant and highly relevant in terms of life safety.

This study, however, will only consider the column elements of the three-storey framed building. The other structural elements, such as slabs, beams, and joint connections, are not factored into this research, despite their potential to also affect the overall stability of the structure.

Literature review

This is evident in the literature, where there has been an emerging interest in designing structures that are durable when exposed to blast loading, especially in the wake of mounting terrorism. Although we may not be able to afford to construct structures that are 100% blast-proof economically, we now have the expertise, equipped with engineering and architectural techniques, that can make a bomb that much less effective. Authors [1] also state that the best solu-

tion is mitigation rather than total resistance; this needs to be achieved through proper threat assessments and vulnerability assessments, which enable informed design decisions based on the most probable types of attacks, such as hand-carried or vehicle-borne bombs. Nevertheless, despite all the screening processes, there are still limitations, and any physical protection strategy needs to consider the residual risks [2].

High explosives undergo rapid chemical reactions that emit intense quantities of hot gas at high pressures and temperatures, producing blast waves that travel at supersonic velocities and may result in severe structural destruction, particularly in closed or confined areas. The blasts commonly cause three categories of loading: air shock waves, dynamic pressure, and ground shock waves, which produce complicated loading patterns initially emerging in the form of overpressure and subsequently with negative pressure periods that cannot be measured accurately [3]. The behaviour of materials (such as steel and aluminium) at these high strain rates is quite different from their behaviour at low, or effectively static, loads. The manner and level of deformation vary with the specifics of the cross-sections, the material structure, and their compatibility with other parts of the connection in the case of composite or hybrid sections.

The distribution of loads in multi-storey buildings requires a good understanding of how loads are distributed and categorised within the building. Columns and walls transmit the vertical loads, primarily caused by gravity, to the base. In contrast, the façades and floor slabs usually resist the horizontal loads caused by wind or blast. Columns with varying cross-sections add complexity to the design, as there are very few exact analytical solutions to their stability, particularly when the loading is a combination of different loads. Structural codes contain some degree of guidance, but differ between countries and between firms when it comes to interpretation [4]. Furthermore, the conventional design approach of the past could not foresee the necessity to safeguard buildings against aerial or bomb blasts without a review of the current safety procedures [5].

Many scholars have developed methods for estimating critical buckling loads, but most of these methods rely on simplifying assumptions. Notably, authors [6] observed that even when stability calculations utilise tables as a source of quick

references, they are typically based on elasticity theory. Researchers often assume an even load distribution and uniform column spacing to simplify calculations [7]. To capture more complex behaviours, researchers have employed advanced computational techniques, including finite element analysis [8] and finite difference methods [9], among others.

Airflow, moisture, and other environmental factors significantly contribute to the structural stability of buildings, particularly in tall buildings. When treating a soil mass, the Eurocode (1991) and other studies by authors [10] have emphasised the importance of accurately estimating soil properties that are highly variable and may cause uncertainties of up to 30 per cent in foundation restraint values [11]. In addition, a structural analysis should include both first- and second-order effects, that is, direct loads and deflections, as identified by authors [12].

The iterative approach proposed by Vianello, although relatively unknown, can provide an effective tool for determining the critical buckling loads of members with non-uniform stiffness. However, to apply it in greater detail, there should be knowledge and access to limited writings [11-13]. As described by the authors [14], geometric nonlinearity may contribute to failures in elastic structures. This phenomenon involves the fact that nonlinear deformation multiplies internal stresses; this effect is even more pronounced in multi-column systems or buildings that utilise seismic isolation bearings, as shear deformation and even tensile buckling are possible [15].

Conclusively, the literature highlights the fact that material failure is not the primary concern in the case of blasts, but rather structural instability. To this end, sophisticated analysis procedures, knowledge of material responses to dynamic loads, and sensible descriptions of boundary conditions and shapes must be integrated into modern structural design. This integrative behaviour is necessary to improve the blast resistance and overall stability of buildings.

METHODOLOGY

This study employs a methodology that combines analytical and computational methods to evaluate the stability of brace-reinforced concrete columns under the combined influence of axial and lateral loads resulting from explosive

impact. To illustrate these two eminent structural analysis methods, the Euler Buckling Method and Vianello Iterative Method have been used to determine the critical buckling loads of columns in a three-story framed building structure.

The materials used in the structural model include steel and reinforced concrete. Universal Beams (UB) and Universal Columns (UC) are steel structures with optimum bending strength and economical design considerations. Use of mechanical properties, including modulus of elasticity, yield stress, and section geometry, was an essential consideration in the selection method for columns.

The researchers used RISA-3D software (version 4.5) to perform the analysis, effectively modelling the structure in 3D, applying blast loads, and analysing the resulting stresses and deflections on the structural system. A combination of multiple boundary conditions, column end conditions, and loads was considered in the study to represent real-world situations.

Euler's Method. Based on the governing mathematical equation of classical beam theory and pin-ended boundary conditions, the Euler buckling load was computed as expressed, where:

$$P_{cr} = \frac{\pi^2 EI}{(kL)^2} \tag{1}$$

where E is the modulus of elasticity; I is the moment of inertia; L is the effective length; k is the factor used as effective length.

The researchers studied various end conditions, including pinned-pinned, fixed-free, and fixed-fixed, and compared the critical loads of each.

Table 1 – First three buckled modes and buckling loads for a pinned-pinned prismatic column

Mode	1	2	3
Mode Shape			
Critical buckling load	$\pi^2 \frac{EI}{L^2} \approx 9.8696 \frac{EI}{L^2}$	$4\pi^2 \frac{EI}{L^2} \approx 39.4784 \frac{EI}{L^2}$	$9\pi^2 \frac{EI}{L^2} \approx 88.8264 \frac{EI}{L^2}$

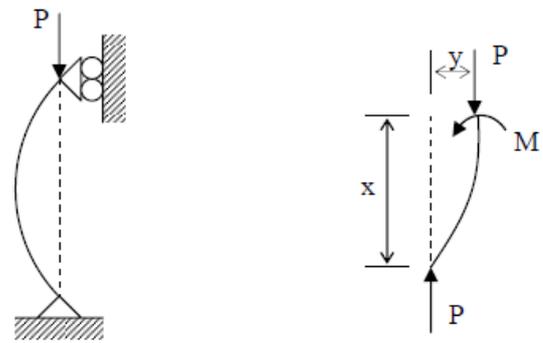


Figure 1 – Pin-ended Euler column

Vianello's Method. The researchers employed the Vianello iterative approach to supplement the analytical method used by Euler, particularly for analysing a non-uniformly stiff column. It is especially applicable to a non-uniformly stiff column. To evaluate successive approximations of lateral deflections numerically, a numerical table was formed by tracing initial guesses of deflection profiles y_a and iteratively adjusting them to convergence with derived deflection y_b .

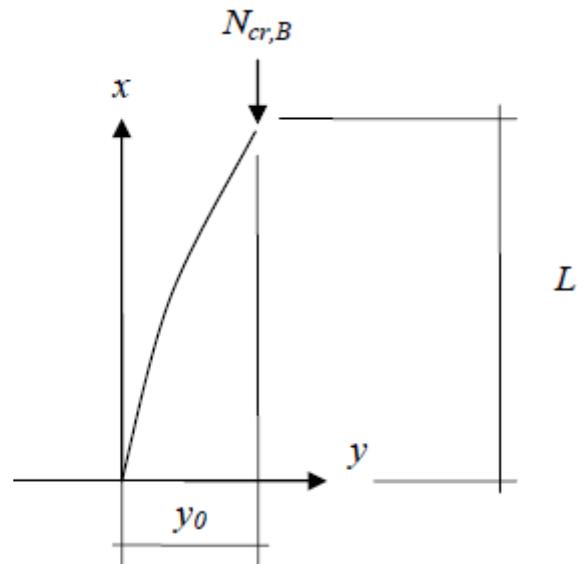


Figure 2 – A column with a vertical force N

Iteration was done through the setting of assumed deflections y_a : a) To calculate curvature and angular deflections y_b and y_a ; b) Updating deflections y_b using former iterations; c) Making sure of the convergence of y_a/y_b ratio with its height. The final converged value was used to get the critical buckling load:

$$N_{cr,B} = K_v \frac{EI}{L_h^2} \tag{2}$$

Table 1 – Final step

$\frac{X}{L}$	ya	y''	y'	yb	$\frac{y_a}{y_b}$
1	1	0	6.36	40.63	0.025
0.9	0.84	0.16	6.20	34.27	0.025
0.8	0.69	0.31	5.89	28.07	0.025
0.7	0.55	0.45	5.44	22.18	0.025
0.6	0.41	0.59	4.85	16.74	0.025
0.5	0.29	0.71	4.14	11.90	0.025
0.4	0.19	0.81	3.33	7.76	0.025
0.3	0.11	0.89	2.44	4.43	0.025
0.2	0.05	0.95	1.49	1.99	0.025
0.1	0.01	0.99	0.50	0.50	0.025
	0	1	0	0	0.000
	Y0	$\frac{N_{cr,B} \cdot Y_0}{EI}$	$\frac{N_{cr,B} \cdot Y_0 \cdot \Delta x}{EI}$	$\frac{N_{cr,B} \cdot Y_0 \cdot \Delta x_2}{EI}$	
				168.46	
Total	4.14				
Δx	0.1				
Ncr,B	2.46(K)	EI/L2			

Blast Pressure Estimation. To model lateral loading due to explosions, the blast wave parameters were obtained by applying known empirical equations and scaling laws (e.g., UFC 3-340-02,

Brode, Mills, and Newmark equations). The researchers varied the corresponding TNT weight and standoff distances, then calculated the overpressures on each floor level.

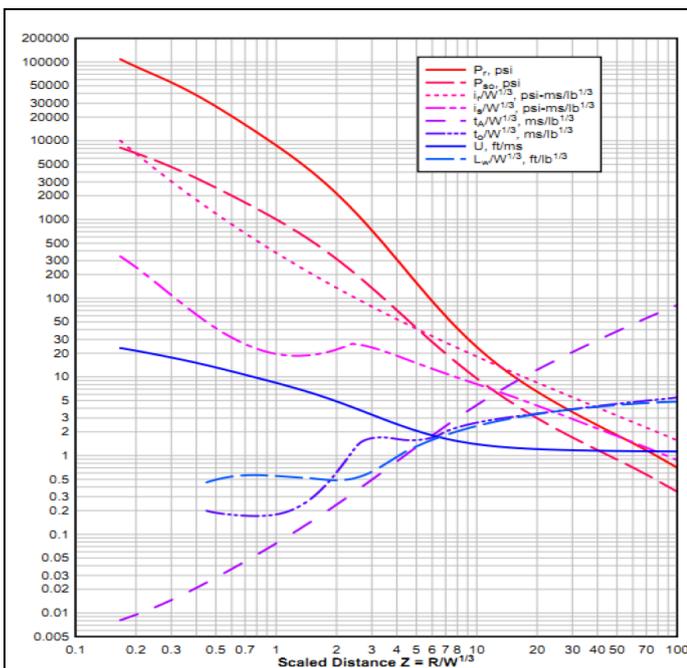


Figure 1 – Positive phase shock wave parameters for a hemispherical TNT explosion on the surface at sea level

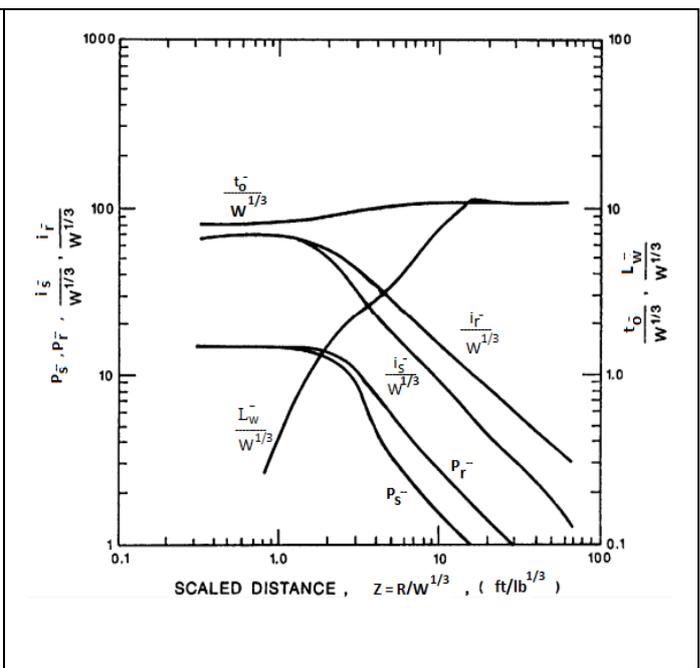


Figure 2 – Negative phase shock wave parameters for hemispherical TNT explosion on the surface at sea level

RESULTS AND ANALYSIS

This chapter presents the computational findings on the stability of column members under combined axial and lateral loading (in the context of a

blast). They also analysed how changing boundary conditions and the geometry of the columns might affect the critical buckling loads of columns, comparing the results obtained by both

the Euler method and the Vianello iterative method.

Vianello Iteration Results. The Vianello method was applied through a seven-step iterative process, starting from assumed deflections (y_a), curvature (y''), angular deflections (y'), and refined deflection outputs (y_b). Convergence of the ratio y_a/y_b confirmed the accuracy of the solution.

a) Final Vianello constant, $K_v = 2.46$, closely matches the Euler constant ($kE = 2$) for cantilever columns.

b) This validates the method's precision in determining critical loads related to bending.

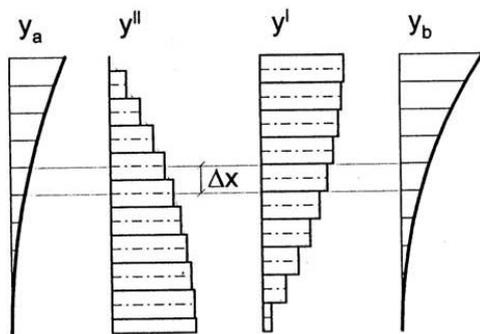


Figure 3 – Curvatures relating to Δx

Euler Buckling Load Evaluation. $P_{crP}_{\{cr\}}P_{cr}$ was calculated on various column profiles whose support was in varied conditions:

Table 2 – Summary of the computed critical loads, indicating that the larger section (356×406×634) maintains significantly higher resistance to buckling across all floors

Column Section	1st Floor (kN)	2nd Floor (kN)	3rd Floor (kN)
838×292×229	9	2	1
356×406×634	23	6	3
305×305×283	2	0.55	0.38

Variation of Blast Pressures with Distance. The analysis output demonstrated that the method used by Euler, although relatively simple in calculation and widely applicable, is conducive to conservative estimates and likely underestimates behaviour during buckling in more complex loading or geometry. Vianello's alternative approach, which requires more computations, is more accurate, especially when dealing with structural

members that have varying stiffnesses throughout their length.

Table 3 – Pressure decay due to distance is evident across all explosive weights

W (kg)	R (m)	Pr (psi)	Pso (psi)
100	20	22	9
100	30	11	4.9
200	20	39	15
200	40	12.8	4.8

Moreover, the research highlighted that loads significantly influence the triaxial behaviour of columns through the cross-sectional geometry of the column, boundary conditions at the supports, and the distance of the explosive source. These results verify the significant value of column design tactics, particularly in hazardous areas or structures prone to blasts.

The values obtained by analysing the columns of different sizes in a three-story construction show that the critical load of buckling decreases as the ground floor moves to the higher floors. This occurs because the columns become longer as the building rises, reducing their rigidity and increasing their susceptibility to buckling. That is to say that the columns that are shorter on the lower levels have higher critical loads, and the columns that are longer on higher floors have greater susceptibility to instability.

Additionally, the researchers observed that the critical buckling load increases with the size and moment of inertia of the column section. Larger sections resist lateral deflection more effectively and remain stable under axial and lateral loading.

Regarding the impact of blast pressure, it has been established beyond doubt that the strength of the blast load a structure will face is highly sensitive to both the standoff distance and the explosive weight. The farther the location is from the point of origin of the blast, the less this overpressure will affect the structure. For instance, in the 100 kg TNT charge scenario, the researchers observed a 50% decrease in blast pressure as the distance increased from 20 m to 30 m, and a further 72.27% reduction as the distance increased from 30 m to 40 m. Similarly, the pressure increased exponentially by 150% when the weight of the explosive was doubled (from 100 kg to 200 kg) at a fixed distance of 20 m.

This discussion clearly illustrates the importance of the R/W ratio (standoff distance to charge weight) dependency on blast parameters. The discoveries play a crucial role in structural blast design, and it is therefore essential that sufficient challenges regarding design considerations are made where there is a high likelihood of blasts. Moreover, it favours the incorporation of proper spacing as well as the optimal column segments in structural design to counteract possible collapse due to explosion forces.

CONCLUSIONS

The paper has demonstrated that the force of blast pressure on building structures is highly sensitive to the weight of the explosive charge and the standoff distance of the explosion source. In particular, an increment in the blast weight by an amount of 100 kg to 200 kg led to a corresponding increment in the peak static pressure (Pso) of about 66.6% and increase to 300 kg of blast weight resulted in the expansion of peak static pressure (Pso) by an amount of 122% at a fixed standoff distance of 20 m. On the other hand, Pso and peak became considerably lower with overpressure (Pr) as the standoff distance developed. For example, reducing the standoff distance from 40 m to 20 m lowered the pressure

values to approximately 65%, based on the weight of the explosives.

The study found that the columns of the ground floor of the building were under greater blast-induced stress than those of any other storey, as they are closer to the source of the blast. In addition, it was established that structures are subject to blasts, and the influence of the waves is on a millisecond scale; therefore, the significance of structural severe dynamic behaviour during rapid response is evident. This correlation between the scaled distance (Z) and the blast pressures was observed to be inversely proportional, as predicted by the theoretical model.

On the whole, the debris emphasised the importance of well-designed columns, especially in buildings of high risk. Engineers and code developers should consider blast effects as a regular part of structural design and practice.

This research focused on investigating column stability in a taller three-storey property structure, considering both axial and lateral loading. The other structural components, including beams, slabs and joints, were not taken into consideration. Additionally, the dynamic interaction between the structural elements and material nonlinearities was beyond the scope of this analysis.

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