Formation of Future Mathematics Teachers' Methodological Knowledge and Skills While Studying Mathematical Analysis at Pedagogical Universities

Nataliia Kuhai [1], **Mykola Kalinichenko** [2]

[1] *Oleksandr Dovzhenko Hlukhiv National Pedagogical University*
24 Kyiv Street, Hlukhiv, Sumy, Ukraine, 41400

[2] *Institute of Radio Astronomy of National Academy of Sciences of Ukraine*
4 Mystetstv Street, Kharkiv, Ukraine, 61002

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Corresponding Author:
Nataliia Kuhai
nkuhai@gmail.com

Abstract. The article presents the results of a comprehensive (theoretical and practical) investigation devoted to observing the possibility and feasibility of forming future mathematics teachers' methodological knowledge and skills while studying mathematical analysis at pedagogical universities. The investigation uses both quantitative and qualitative methods. Qualitative research methods include observations, conversations, and surveys of students about the motives for choosing a profession, motives and needs for mastering knowledge and skills, including methodological. Testing and questionnaires are quantitative methods. During the study, the authors have established that while studying mathematical analysis, there are vast opportunities for forming methodological knowledge of all levels (philosophical, general scientific, specifically scientific, technological) and relevant methodological skills, which should be combined into four groups: general methodological; mathematical and methodological; organisational and methodological; communicative and methodological. The study highlights that the formation of methodological knowledge and skills should be conducted systematically, comprehensively, continuously and gradually. The study involved 94 higher education students clustered into experimental and control groups. Learning in the control group was conducted traditionally, whereas learning in the experimental group was based on the methodology developed by the authors. The main provisions of the mentioned technique are the selection of methodological knowledge and skills for each topic of mathematical analysis, the selection and rational combination of innovative and traditional forms of learning, methods of conventional and interactive understanding of mathematical analysis, and the selection of appropriate material and ideal teaching aids. Based on the conducted questionnaire, testing, and processing of research data, the efficiency of the offered technique for forming methodological knowledge and skills during the study of the mathematical analysis has been proved.

Keywords: methodological knowledge; methodological skills; mathematical analysis; future mathematics teachers; pedagogical university; STEAM education.

INTRODUCTION

The current state of society's development is characterised by a rapid increase in the information flow and the rise of the importance of mathematical knowledge in humanity's activities. Consequently, training a person familiar with the research methods of various objects involving mathematical knowledge, skilled in specific mathematical tools, techniques and technologies for data collection, analysis and display, and capable of producing new knowledge becomes critical. In other words, a specialist with methodological knowledge and skills is required.

Authors [1-3] and others stressed the importance of methodology and methodological knowledge and skills in the educational process.

The content of the vast majority of future mathematics teachers' methodological knowledge and skills correlates with the essence and content of the functions of STEM education authors [4-7] and abilities of the XXI century, the content and structure of which are defined in the documents of the educational association "The Partnership for 21st Century Learning (Skills)" (P21) [8, 9]. They include understanding the role of mathematics in today's world, learning the fundamental concepts of mathematics and having the skills to apply mathematics to engineering and other scientific and practical areas by authors [7]; integration of knowledge both within the discipline and between disciplines by authors [4]; students' understanding of mathematical and natural concepts and their usage by authors [5], etc. Therefore, acquiring methodological knowledge and skills from future mathematics teachers helps prepare students for the implementation of STEM education, employment, and self-realisation.

The future teachers' methodological knowledge and skills, along with cognitive and operational components of their methodological competence, were considered in the works by authors [10-12] etc. Researchers such as authors [13, 14] studied methodological knowledge as a cognitive component of the teacher's methodological culture.

The methodological culture of future teachers, including mathematics teachers, is the primary purpose of scientists' methodological training authors [13, 15, 16]. The authors' works considered the formation and development of methodological training for future teachers of various subjects [17, 18].

In the structure of methodological training of future teachers, the study distinguishes three components [15]: motivational (formation of motivational and holistic personal attitude to professional activity, in particular, to methodological activity); substantive (mastering methodological knowledge); operational (mastering methodological skills).

Since the issues of formation of future mathematics teachers' methodological knowledge and skills while studying mathematical analysis in today's conditions have not been comprehensively studied, the article aims to show the feasibility and possibility of forming methodological knowledge and skills of all levels in future mathematics teachers and to check the effectiveness of the proposed method for this process.

The study's objectives are to indicate the methodological knowledge and skills that should be formed in future mathematics teachers while studying mathematical analysis, to demonstrate ways to develop this knowledge and skills and to check the effectiveness of the proposed method for this process.

Theoretical background

The methodology concept is multifaceted and complex; various scholars have attributed different meanings to this concept. Scholars mainly discuss the methodology of a particular science or research. Author [19] views methodology as a general research strategy that outlines the research path. Authors' work [20] states that methodology provides a general theoretical view of knowledge and its production. Author [21] examines the methodology of science and emphasises that it «describes or prescribes the methods used in the construction, testing and justification of scientific hypotheses and theories, as well as the epistemic and cognitive values that (should) guide scientists in their research». Authors [22] see methodology as the study of the organisation of any productive human activity (scientific, educational, research, and practical professional activity, etc.). In the study, the methodology of a specific activity is understood as the study of the organisation of this activity, and the methodology of a particular science (mathematics, chemistry, physics, etc.) is considered as the study of the organisation of the corresponding scientific activity. The methodology structure under this approach remains the same in form but differs in content for various productive activities, including students' educational and cognitive activities.
By the future mathematics teachers' methodological knowledge, which can be formed while studying mathematical disciplines, including mathematical analysis, the authors mean knowledge

about the nature, methods of recording and constructing knowledge, knowledge about knowledge and techniques of cognition, as well as knowledge about the organisation of productive activity author [23]. Methodological skills are considered the ability to apply methodological knowledge. Similar to the levels of methodology proposed by the author [24], it is reasonable to identify four levels of methodological knowledge: philosophical, general scientific, specifically scientific, and technological. During the study of mathematical analysis, there are opportunities for illustrating and forming methodological knowledge of all levels. While selecting elements of methodological knowledge, the content filling of corresponding levels of methodology in scientific activity in mathematics has been focused on. To distinguish essential features of each element of methodological knowledge, the authors have relied on the content of the mentioned elements (law, principle, category, concept, method, etc.).

Philosophers should mention knowledge about the following philosophical categories among the methodological knowledge at the intellectual level:

1) Existence and uniqueness;
2) Finiteness and infinity;
3) Content and form;
4) General and concrete (partial, singular);
5) Discreteness and continuity;
6) Logical and historical;
7) Cause and effect.

In mathematics, particularly in mathematical analysis, there is an opportunity to enrich knowledge about philosophy's laws, principles, and categories. For example, the law of transformation of quantitative into qualitative changes can illustrate the emerging formation and development of the infinitesimal calculus. The principle of determinism is manifested through the functional connection. In the simplest case, the functional relationship between individual properties or characteristics of the object is expressed through the function, which is the subject of mathematical analysis.

Methodological knowledge of the general scientific level is, as a rule, knowledge about methods of proving statements, methods of mathematical induction, methods of complete induction, analogy method, methods of mathematical modelling, methods of generalisations, methods of similarity, method of formalisation, method of idealisation, computational experiment, algorithmic method, coordinate method, vector method, and algebraic methods. This level of knowledge also includes knowledge about general scientific terms (theory, concept, definition, axiom, theorem, lemma, criterion, consequence, hypothesis, principle, property, law, rule, theoretical statement, quantity, etc.), mathematical symbols, and heuristic schemes.

The methodological knowledge of the specifically scientific level will encompass understanding the subject of the disciplined study, specifically scientific research methods; fundamental (the most important) concepts including basic (undefined) concepts; fundamental (the most important) relations including the basic relations between the basic concepts; fundamental (the most critical) theoretical facts (statements) including definitions, axioms, lemmas, theorems, consequences, rules, etc.; connections with other disciplines of the mathematical cycle; limits of knowledge applicability; and history of emergence and development of scientific theory related to this discipline. Among the methodological knowledge of the technological level, which should be formed on the material of mathematical analysis, it is distinguished the knowledge about using computer tools of mathematics, namely knowledge about the availability of such tools (their names, access); ease of use; computer capabilities; direct usage of the tools; and the need and feasibility of using a computer tool.

The author reveals the content of the listed methodological knowledge in more detail [25]. Considering that the theoretical basis of methodological skills is multilevel methodological knowledge, the study proposes to consider four groups of methodological skills: general methodological, mathematical and methodological, organisational and methodological, and communicative and methodological. For instance, the list of skills for each group is provided in the work by authors [26]. Some examples of those methodological skills that are possible and appropriate to develop while studying mathematical analysis are the below-mentioned.

General methodological skills are connected with mastering the universal methods of cognition and transformation of reality: to investigate the issues of existence and uniqueness; to recognise finite (infinite) sets and to justify the choice; to establish cause and effect relationships; to see specific in general; and to establish interdisciplinary links, etc.

Mathematical and methodological skills are applying general mathematical methods and methods specific to mathematical analysis, including the limiting method. When studying mathematical analysis, there are all possibilities for the formation of skills to apply methods related to the methodology of general scientific (comparison, mathematical modelling, generalisation, complete induction, etc.) and specifically scientific (methods of function research, methods of finding limits, derivatives, integrals, etc.) levels. This group of methodological skills includes identifying fundamental concepts and facts of a particular topic (content module) and establishing relationships between them.

Organisational and methodological skills are considered the ability to self-organise productive activities (it is mainly educational and cognitive, research and practical for future mathematics teachers): analysis and planning of activities; search for mathematical information; organisation of work with various mathematical objects, such as concept, mathematical fact, problem, etc.; and ability to select and apply computer mathematics software, etc.

Communicative skills, formed while studying the disciplines of mathematical direction, particularly mathematical analysis, constitute a group of communicative and methodological skills: the knowledge of the mathematical language and native language as specific ways of communication and the usage and transformation of the system of sign-symbolic means of mathematics.

Forming future mathematics teachers' methodological knowledge and skills while studying mathematical analysis involves the phasing of this process: propaedeutic, educational and active, evaluative and reflexive. At the propaedeutic stage, students are acquainted with the content of methodological knowledge and skills, systematisation of previously acquired (in-school course) methodological knowledge, and preliminary preparation for developing new methodological knowledge and skills and their application. The educational and active stages include mastering methodological knowledge and skills, applying them typically by example, and clarifying the limits of applicability of the acquired knowledge and skills. At the evaluative and reflexive stage, students' high level of independence and creativity in acquiring and using methodological knowledge and skills, transferring this knowledge and skills in new conditions, and reflecting are envisaged. Students attempt to find answers to questions such as "What do I know?", "What can I do?", "Why do I need this as a future mathematics teacher?".

It should be noted that the proposed stages of formation of future mathematics teachers' methodological knowledge and skills are different from the stages of students' learning; this means that the years of students' study (undergraduate or graduate, 1st year or 4th) do not directly correlate with the proposed stages. The duration of each stage for different methodological knowledge and skills varies and can be realised while studying not just one mathematical discipline.

The formation of methodological knowledge and skills while studying mathematical analysis should be conducted systematically, systemically, purposefully, and in organic unity with the formation of content knowledge and skills. For this purpose, it is advisable to identify each topic as a set of methodological knowledge and skills by the mentioned levels and groups. Particular emphasis should be placed on methodological knowledge and skills that are professionally relevant for future mathematics teachers and will contribute to the correct solution of methodological issues that arise in teaching mathematics at school. Table 1 shows a fragment of our set of future mathematics teachers' methodological knowledge in mathematical analysis.

Table 1 – A fragment of a set of methodological knowledge in mathematical analysis

Title of the topic	Methodological knowledge
Function	*PhL:* the principle of unity and diversity of the world (object, property, relationship), the principle of determinism (cause, effect, cause and effect connection, the causality conditions), the principle of development (the law of transition from quantitative to qualitative changes), uniqueness, existence, the criterion of truth in mathematics. *GSL:* mathematical modelling, analogy, method of mathematical induction, classification, idealisation, generalisation, theory structure, mathematical symbols and signs.
Title of the topic	Methodological knowledge
Function	*SSL:* subject of research, elementary math-

Title of the topic	Methodological knowledge
	ematics methods for function research, fundamental concepts, facts, relations between them, interdisciplinary connections, history of function doctrine development. *TL:* knowledge of preparation, design and defence of abstract projects; knowledge of mathematics software for graphing functions.

Notes: PhL – philosophical level of methodological knowledge; GSL – general scientific, SSL – specifically scientific, TL – technological

Educators should perform the diagnostics of future mathematics teachers' methodological knowledge and skills using a complex approach. In order to achieve this, we have created a system of criteria, indicators, and levels for the formation of future mathematics teachers' methodological knowledge and skills.

As criteria, the authors have singled out the following: motivational (students' attitude to the future teaching profession; students' motivation to acquire this profession; the presence of motives and needs in the formation of methodological knowledge and skills; attitude to educational activities), cognitive (students' mastery of methodological knowledge and skills; mobility of future mathematics teacher's methodological knowledge and skills, the quality of execution of tasks aimed at the formation of methodological knowledge and skills), active and operational (possession of a set of methodological skills, systematic acquisition of methodological knowledge and skills), reflexive (ability to assess the formation of methodological knowledge and skills, ability to self-esteem and reflection). According to the degree of manifestation of the criteria, the study has identified four levels of formation of the future mathematics teacher's methodological knowledge and skills: entry, middle, sufficient, high (the model of forming methodological knowledge and skills while studying mathematical analysis is shown in Figure 1).

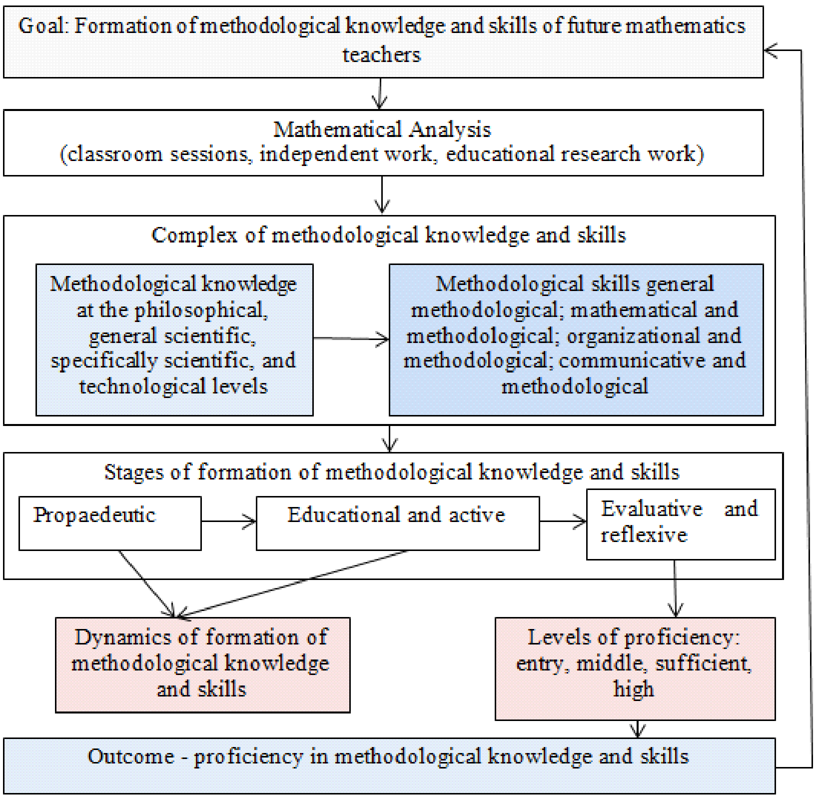

Figure 1 – The model of forming methodological knowledge and skills

METHOD

Research Design. To achieve the goal and solve the set tasks, the authors have used theoretical, logical and empirical methods and techniques of research: analysis and synthesis, comparison, generalisation, methods of mathematical statistics, observation, interviews, surveys, questionnaires, testing, methods of traditional and interactive learning, and ascertaining, search and formative stages of the pedagogical experiment.

Participants. The investigation involved 94 students in the 1st and 2nd year of bachelor's degree, studying in the subject speciality 014 Secondary Education (Mathematics), five lecturers of mathematical analysis (Berdyansk State Pedagogical University, Oleksandr Dovzhenko Hlukhiv National Pedagogical University, Anton Makarenko Sumy State Pedagogical University, Pavlo Tychyna Uman State Pedagogical University), 11 teachers of Mathematics of general secondary education institutions of Hlukhiv town.

Ethical Issues. All participants of the study voluntarily participated in the experiment. The researchers informed them in advance about the stages of the study, its content, and the expected results. The researchers obtained informed consent from all participants. The study adhered to objectivity, fairness, anonymity, and confidentiality principles. The presented research was positively evaluated and approved by the Committee on Ethics and Academic Integrity of Oleksandr Dovzhenko Hlukhiv National Pedagogical University, with file No M20220209.

Data Collection and Data Analysis Tools. The collection of research data included the use of questionnaires (to assess the manifestation of motivational and reflective criteria), tests (current and control), and control works (to determine the level of formation of methodological knowledge and skills). The researchers calculated quantitative indicators (the corresponding coefficients) to diagnose the level of formation of methodological knowledge and skills according to the criteria mentioned above. Assuming that the identified criteria are equivalent in assessing the level of formation, the level of formation of methodological knowledge and skills of each student was defined as the arithmetic mean of all numerical values of the calculated coefficients. Excel was used to process quantitative data.

Pearson's test was used to test the hypothesis of a standard sample distribution, student's test and Fisher -Snedekor test were used to test the homogeneity of the samples, and Fisher's criterion (Fisher's angular transformation) was used to test the reliability of the obtained data.

Procedure. The control test consisted of 3 parts and was conducted to select control and experimental groups. It had the following structure of it was as follows:

Task 1.1 – 1.10 involved choosing one correct answer out of four options provided;

Task 2.1 – 2.2 required writing down the correct answer;

Tasks 3.1–3.2 needed to include a full justification in their solution. All tasks were based on the school mathematics curriculum, including Task 3.1 – solving an exponential equation with a parameter, and Task 3.2, which required demonstrating initial skills in applying the method of mathematical modelling. The instructors selected tasks to estimate not only the level of academic achievement of first-year students but also to obtain subjective results for diagnosing elements of their methodological knowledge and skills. For the convenience of converting the results into the generally accepted scale of assessment at higher education institutions and establishing the level of students' achievement, the test was evaluated at 100 points. The researchers constructed interval distributions and computed sample characteristics for both the control and experimental groups based on the control test results.

$\overline{x}_{EG} = 60.8;\ \overline{x}_{CG} = 60.1;\ \sigma_{EG} = 15.0; \sigma_{CG} = 16.2;$

Corrected variances $S_{EG}^2 = 229.5; S_{CG}^2 = 267.2$

Pearson's criterion was applied to confirm the hypothesis of the normal distribution of samples.

$\chi_{EG}^2 = 1.06 < \chi_{crit}^2(0.05;5) = 11.1$,

$\chi_{CG}^2 = 1.63 < \chi_{crit}^2(0.05;5) = 11.1$

– the hypothesis was confirmed.

To assess the homogeneity of samples, the student's criterion

$t_{observ}=0.21< t_{crit}(0.05;92)=1.96$

– sample means differ insignificantly,

and the Fisher-Snedecor criterion

$F_{observ}=1.16<F_{crit}(0.05;48;44)=3.63$

– variances that differ insignificantly were used.

The teaching of mathematical analysis in the control groups was conducted using the traditional method without emphasising the students' attention to methodological knowledge and skills. The researchers developed appropriate methodological support to organise the teaching of mathematical analysis in experimental groups. Firstly, the authors prepared a complex of methodological expertise that can be formed based on mathematical analysis. This complex also indicates what methodological knowledge and skills are required to form while studying a particular topic. Secondly, it was created: means for activation of students' educational and cognitive activity at lectures (tables, presentations, printed editions, etc.); tasks to determine the level of formation of students' methodological knowledge and skills; a list of abstract topics, projects to test students' organisational skills as well as communicative and methodological skills; methodical instructions for teachers concerning the organisation of training process in the experimental conditions.

At the first lecture on mathematical analysis, the lecturer, with the student's active participation, determined the subject of mathematical analysis, began to highlight the professional significance of this discipline and familiarised with the main methods of cognition, indicating connections with other educational disciplines (in the process of studying the discipline, students specified these connections). At the end of the lecture, the main stages of the development of mathematical analysis were presented (while studying each topic, this information was enriched by the students and presented in the form of essays or projects).

Before studying each topic of mathematical analysis, a complex of methodological knowledge and skills was identified according to the abovementioned levels and groups (Table 1), which should be formed in students while studying this topic. The researchers rationally combined innovative and traditional forms and methods of conventional and interactive teaching of mathematical analysis for this process. There was also a strong focus on the means of learning (tangible and ideal), including ICT: the following methods were most frequently utilised: lecture discussion for elucidating the theoretical foundation of a particular method of cognition, its applicability limits, and its presence in the school mathematics curriculum; group work for constructing structural-logical schemes of methodological knowledge and skills within the scope of a topic (chapter); "microphone" and "complete the thought" for activating previously acquired methodological knowledge and formed methodological skills to encourage learners to hypothesise about the applicability of the knowledge in new conditions; "wide circle discussion" for revealing the essence of new fundamental concepts and propositions; and "brainstorming" for developing skills to clarify relationships between essential concepts and facts, establish cause-and-effect relationships, generalise, perceive the specific within the general, etc. It is also advisable to employ a combination of interactive teaching methods, such as: "method of control questions", "complete the sentence", and "microphone" (for diagnosing the level of proficiency in applying a particular method, for example, the ability to prove statements by the method of mathematical induction). For material visualisation, presentations were mainly used (Microsoft PowerPoint) and the educational software tool GRAN.

There are some specific examples provided. The content of the ability to clarify the structure of mathematical theory as its component includes the ability to distinguish fundamental concepts, facts and relationships of a separate topic, content module, and mathematical analysis. The authors demonstrate the method of forming this skill in the example of a lecture on the topic "Continuity of a function of one variable". At the beginning of the lecture, the instructor introduced the students to the lecture plan. While revealing each issue of the lecture, students' attention was focused on the following questions (using the technology "Discussion of the problem in a general circle"): What concepts were considered? Which of them are new to you in the course of mathematical analysis? What statements were formulated? At the end of the lecture, a summary was presented to answer the following questions: What mathematical objects were considered? Which ones are new to you? Name the properties of the considered mathematical objects. As homework, students were asked to note a list of this topic's most essential concepts and facts. To reveal the essence of new ideas and statements, the instructor discussed the answers to the following questions with the students:

1) What are the elementary steps of checking the continuity (one-sided continuity) of a function at a point discontinuity of a function at a point?

2) Is it possible that the function is neither continuous nor discontinuous at a given point?

3) Can the sum of two discontinuous functions be continuous?

4) Is a composite function necessarily discontinuous if the outer and inner functions are discontinuous?

5) In the definition of the continuity of a function at a point in the language "epsilon - sigma", what does "sigma" depend on?

6) Is every function continuous at an interval bounded at this interval, too?

7) Can a function discontinuous on a segment be bounded on this segment? (The list of all questions is given by authors [27].

The ability to establish interdisciplinary links, especially with a school mathematics course, should be considered from the following positions:

1) Links with the disciplines of the mathematical cycle at the level of concepts and facts (what knowledge and skills are necessary to study this discipline and what knowledge and skills from this discipline will be used to study the following disciplines of the mathematical cycle).

2) Connections at the level of methods of cognition (general methods).

3) Links with other branches of science, technology, and production (application).

The teacher informed students about these connections at the propaedeutic stage of forming methodological knowledge and skills. If the teacher used the method for the first time, they would emphasise which other disciplines (or topics) this method could apply to. At the educational and active stage in lectures (or practical classes), the teacher, together with students, found the answers to these questions using the method of control questions:

1) What the logical gaps in the school mathematics course eliminate the studied facts and methods?

2) What discipline do the used facts belong to? What section? (For example, to solve the problem of a tangent to a plane curve, you need knowledge about the equation of a line with an angular coefficient (analytical geometry, line on the plane) to derive the formula for replacing variables in the triple integral you need knowledge about determinants (linear algebra), etc.).

3) What discipline did you learn about this method while studying?

4) Give some examples of the results of applying the studied facts and methods in various science, technology, and production fields. Investigate the essence of the application of the mentioned facts and techniques.

At the evaluative and reflexive stages, the students developed the ability to establish connections with other branches of science, technology, and production. The students expressed their opinions about the role and importance of mathematics in society. Students' research works played a significant role at this stage: writing essays (mostly 1st year) and educational and research projects. For example, at the end of the section "Function" (1st semester), students were offered to write an essay "Function as a Mathematical Model in Physics (Economics, Biology) Problems" (optional). Students performed the educational and research project "Interdisciplinary Connections of Mathematical Analysis" at the end of the 2nd semester. For this purpose, students were clustered, and each group addressed one of the following questions: mathematical analysis and linear algebra, mathematical analysis and analytic geometry, mathematical analysis and elementary mathematics.

The current and control tests were conducted to determine the formation level of methodological knowledge and skills, for which all three stages of formation were implemented while studying mathematical analysis. The formation level was determined by the results of control tests conducted at the third stage (evaluative and reflexive) of the formation of methodological knowledge and skills. The results of students' independent extracurricular work aimed at forming methodological knowledge and skills, such as preparation and defence of abstracts and projects, compilation of chronological tables of mathematical analysis, execution of individual works, etc., were considered. Control tests were selected to reflect the cognitive criterion and active and operational one for the levels of formation of future mathematics teachers' methodological knowledge and skills.

Extensive methodological knowledge and skills are formed while studying mathematical analysis. After interviews with experienced teachers and an analysis of the author's previous research results, the study identified methodological knowledge and skills that are the most important for future mathematics teachers. They include the method of mathematical induction (MI), the method of mathematical modelling (MM), the limiting method (LM), the methods of function investigation (FI), the technique of derivative calculation (DC), the methods of integration (MI), the method of undetermined coefficients (UC), knowledge of interdisciplinary links and the ability to establish them (IL). The researchers determined the coefficient of knowledge acquisition and skills development for all methods for each student.

While studying mathematical analysis, the first two stages (propaedeutic, educational, and active) of the investigated process are implemented for most components of methodological knowledge of philosophical and general scientific levels and their corresponding methodological skills. To estimate the dynamics of the formation of methodological understanding of philosophical and general scientific levels and the corresponding methodological skills, researchers calculated the coefficient of completeness of knowledge (skills) formation.

$$K = \frac{\sum_{i=1}^{n} n_i}{n \cdot N}, \quad (1)$$

where *n* is the number of elements of knowledge (skills) that must be mastered at a particular stage, n_i is the number of aspects of knowledge (skills) that the i-th student has learned, *N* is the number of students who participated in the experiment [28].

For an individual student, this coefficient was determined by the formula:

$$\bar{K} = \frac{n_i}{n}. \quad (2)$$

To estimate the changes achieved in comparison to the initial level and to clarify the dynamics of formation, researchers calculated the coefficient of development of knowledge (skills):

$$\gamma = \frac{\bar{K}_2}{\bar{K}_1}, \quad (3)$$

where $\bar{K}_1$, $\bar{K}_2$ the coefficients of completeness of knowledge (skills) formation were determined earlier.

Suppose γ >1 the relevant expertise (skills) are formed successfully. The coefficient of development of methodological knowledge (skills) at the philosophical and general scientific levels:

$$\gamma = \frac{\bar{K}_2}{\bar{K}_1}. \quad (4)$$

It was not included in the formulas for calculating the level of proficiency in students' methodological knowledge and skills. The mentioned coefficient served only as an indicator of the dynamics of the formation of such knowledge among future mathematics teachers while studying mathematical analysis.

In order to determine the degree of manifestation of motivational and reflective criteria, students conducted questionnaires at the end of the 4th semester (all control sections were conducted before that; students passed the exam in mathematical analysis). The questionnaire consisted of 30 questions, starting with "To what extent do you: ....". A student chose the answer to each question from five possible answers: fully enough (a); sufficiently (b); I can't say (c); insufficiently (d); not enough (e). Here are some examples of questions: "To what extent do you: 1) evaluate your future profession as your vocation; 2) realise the social significance of the teaching profession; 3) realise the need to apply methodological knowledge and skills in the study of mathematical analysis". According to the results of the questionnaire, the coefficient of manifestation of these criteria was calculated:

$$K_{MR} = \frac{a + 0.5b + c - 0.5d - e}{30}, \quad (5)$$

According to authors [29], where a, b, c, d, and *e* are the numbers of relevant items on the scale selected by the student, K_{MR} is the coefficient of manifestation of motivational and reflexive criteria.

The level of formation of methodological knowledge at the technological level and corresponding methodological skills was determined by the students' results of implementation and presentation of educational and research works (abstracts, projects). For this purpose, the coefficient of skill formation was calculated:

$$K_{TL} = \frac{n_i}{n}, \quad (6)$$

where n – is the number of skills that need to be mastered at a particular stage, n_i – is the number of skills learned by the i-th student.

At the end of the study, the researchers determined the coefficient of the formation of methodological knowledge and skills for each student using the formula:

$$\overline{K} = \frac{K_{MR} + K_{TL} + K_{MMI} + K_{MM} + K_{LM} + K_{FI} + K_{DC} + K_{MI} + K_{UC} + K_{IL}}{10}, \quad (7)$$

Here, the sub-indices indicate the names of the method, criterion or level of methodological expertise for which the coefficient of formation of knowledge and skills was determined. Following the value of the coefficient, we established the level of formation of students' methodological knowledge and skills:

If $0.9 < \bar{K} \leq 1$ the level is high and sufficient, $0.5 < \bar{K} \leq 0.7$ the is a middle-level $\bar{K} \leq 0.5$, and the level is an entry.

Fisher's criterion (angular Fisher transform) was used to verify the validity of the obtained data.

RESULTS AND DISCUSSION

It was noted above that for most elements of methodological knowledge of philosophical and general scientific levels and their corresponding methodological skills while studying mathematical analysis, two stages (propaedeutic and educational and active) of forming methodological knowledge and skills are implemented. To determine the dynamics of the formation of such methodological knowledge and skills, we determined the coefficient of knowledge (skills) development:

$$\gamma = \frac{\bar{K}_2}{\bar{K}_1}. \quad (8)$$

The composition of the elements of knowledge (skills) that must be mastered at a particular stage of the study of mathematical analysis was previously determined to calculate the coefficient. For example, to calculate the coefficient of knowledge about the principle of uniqueness and diversity of the world, the authors have identified the following elements of knowledge: 1) mathematical object; 2) property of a mathematical object; 3) the connections of a mathematical object with other objects; 4) ideal and material.

According to this principle, knowledge was Diagnosed after studying most topics and content modules specified in the developed methodological expertise. To do this, students were asked to perform the following tasks:

1) Specify the mathematical objects that were first introduced while studying the topic (content module) (the name of the topic or module).

2) Among the proposed statements, select the properties of a particular mathematical object (the name of the mathematical object is given; the proposed statements were usually four, and the correct answers could be from 0 to 4).

3) Name a more general mathematical object for a given mathematical object; name the mathematical object for which the object is a generalisation.

4) Provide some examples of material objects for which a given mathematical object is an idealisation.

5) Formulate the problems that led to the introduction of this mathematical object.

For diagnostics of methodological knowledge and skills, the time allocated for independent work of students in the discipline is used. Table 2 shows the results of serial sections in the experimental and control groups, conducted at the end of the 1st and the end of the 4th semesters.

Table 2 – Dynamics of formation of methodological knowledge and skills (EG i CG)

Methodological knowledge	EG			CG		
	$\overline{K}_1$	$\overline{K}_2$	γ	$\overline{K}_1$	$\overline{K}_2$	γ
Uniqueness and diversity of the world	0.54	0.63	1.17	0.51	0.57	1.12
Principle of development	0.43	0.55	1.28	0.35	0.41	1.17
Principle of determinism	0.48	0.62	1.29	0.41	0.49	1.20
The main categories of dialectics (existence, uniqueness, finite, infinite, continuity, discreteness, logical, historical, rational, irrational, content and form, general, separate)	0.56	0.72	1.29	0.34	0.39	1.15
Structure of the theory	0.63	0.71	1.13	0.33	0.37	1.12
Sign-symbolic means of mathematics	0.54	0.72	1.33	0.45	0.51	1.13
Methods of empirical research (comparison, mathematical experiment)	0.45	0.54	1.20	0.38	0.41	1.08
Methods of theoretical cognition (formalisation, idealisation, abstraction, generalisation, induction, deduction, analogy, mathematical modelling)	0.56	0.74	1.32	0.34	0.35	1.03

To determine the level of formation of future mathematics teachers' methodological knowledge and skills while studying mathematical analysis, the authors calculated ten coefficients of formation of relevant knowledge and skills (mentioned above).

Since it is impossible to combine tasks for diagnostics of the listed knowledge and skills into one control slice, theoretical and practical content tasks were selected for each element of methodological knowledge and the corresponding ability to apply them. There are some examples.

The ability to establish interdisciplinary links includes the following elements:

1) Name the knowledge from other mathematical disciplines that were necessary to study the topic (the content module);

2) Name the skills that were formed while studying other mathematical disciplines and which are necessary to examine the topic (the content module);

3) Name the mathematical disciplines in which the knowledge of this topic (this content module) was used while studying;

4) Name the mathematical disciplines in which the skills of this topic (this content module) were used while studying;

5) Indicate the disciplines that have standard methods with the discipline under study;

6) Name the branches of science, technology, and production where you can apply the concepts and facts of the topic (the content module).

To diagnose the development of this skill, students are asked to answer questions after studying a topic (content module). The coefficient for each student was calculated using the formula:

$$\bar{K} = \frac{n_i}{n}. \quad (9)$$

To diagnose students' knowledge about the method of mathematical induction (MI) and their ability to apply it, we offered them the following tasks:

1) Name the theoretical basis of the MI method;

2) describe the statements for which the MI method should be used;

3) Create a guideline diagram for proving statements by using the MI method;

4) Give an example of 3 statements from the course of mathematical analysis, which are proved by using the MI method;

5) Prove that

$$\forall n \in N \ \frac{1}{1\cdot 3} + \frac{7}{3\cdot 5} + \ldots + \frac{2n^2-1}{(2n-1)(2n+1)} = \frac{n^2}{2n+1};$$

6) Prove that $\forall n \in N, n > 4 \ 2^n > n^2$;

7) Prove that $\forall n \in N \ (7^n - 1) \vdots 6$.

This paper is worth 13 points: each theoretical question is worth 1 point, and the practical question is worth 3 points.

For each student, the coefficient of knowledge acquisition and skills development was determined by the formula:

$$K_{MMI} = \frac{i}{13}, \quad (10)$$

where i – is the number of points scored by this student.

Similarly, the coefficients of knowledge acquisition and skills development were calculated in other cases.

The distribution of students of the experimental and control groups according to the levels of formation of methodological knowledge and skills are presented in Table 3 and in Figure 2 (the number of students of the corresponding level is marked on the vertical axis).

Table 3 – The distribution of students according to the levels of formation of methodological knowledge and skills (at the end of the experiment)

Group	Entry level $\overline{K} \le 0.5$		Middle level $0.5 < \overline{K} \le 0.7$		Sufficient level $0.7 < \overline{K} \le 0.9$		High level $0.9 < \overline{K} \le 1$	
	number	%	number	%	number	%	number	%
EG: $n_x = 45$	5	11.1	11	24.4	22	48.9%	7	15.6
CG: $n_y = 49$	9	18.4	21	42.8	15	30.6%	4	8.2
Together: $n = 94$	14		32		37		11	

Based on the statistical φ* Fisher criterion, it was concluded that the level of formation of methodological knowledge and skills in the experimental group is higher than in the control group (calculated $\varphi^*_{emp} = 2.51$; for the significance level $\rho = 0.05$ the critical value of the criterion $\varphi^*_{crit.} = 1.64$; $\varphi^*_{emp} > \varphi^*_{crit}$).

At the current stage of societal development, the content of professionalism for future teachers, including mathematics teachers, encompasses qualitatively new requirements: the ability to organise the educational process at a high level, conduct independent scientific research, master new technologies and information systems, generalise advanced methodological experience, critically evaluate the results of their work, creatively engage in self-improvement, and pursue lifelong learning, etc. We can ensure the implementation of these requirements if future teachers develop methodological competence and methodological culture. Therefore, future mathematics teachers' methodological knowledge and skills indicate their development.

The mathematics teacher should not only possess a high level of methodological knowledge and skills but also be capable of fostering the development of elements of methodological knowledge and skills in students, including recognising real-life or subject-related situations as problems that can be solved using mathematical methods; formulating them in mathematical language; applying mathematical models in the study of natural and other subjects; analysing, comparing, predicting outcomes, generalising, and systematising mathematical objects based on specific properties; using appropriate sources to search for mathematical information, independently analysing it, and conveying the mathematical essence in textual, graphical, tabular, symbolic forms, etc. [30]. Furthermore, the formation of methodological knowledge and skills, especially methodological knowledge at the technological level and organisational and methodological skills, will contribute to increased activity in students' educational and research activities and the quality of student research projects.

Mathematical analysis, as one of the components of the educational and professional training program for future mathematics teachers, occupies perhaps the most critical place in this process. This discipline is essential for the student's future professional activity (many issues in the school course of mathematics are theoretically substantiated while studying mathematical analysis). An extensive array of mathematical concepts, statements, connections between them, and a vast arsenal of cognition methods of different methodological levels contribute to the effective formation of future mathematics teachers' methodological knowledge and skills. Students should be taught methodological knowledge gradually and purposefully; this coincides with the authors' conclusions [3, 25]. Knowledge of the methodology of mathematics contributes to a better understanding of mathematics. It fills students with a sense of satisfaction from studying mathematics, regardless of the level of academic achievement authors [1]. The results of this study confirmed this.

The results presented in Table 2 show that the methodological knowledge of the philosophical and general scientific levels and the corresponding methodological skills were successfully formed while studying mathematical analysis in the experimental and control groups. For students of the experimental group, in most cases, the coefficients of knowledge acquisition and skills formation and the coefficient of development are higher than for the students of the control group. It gives grounds to state that the proposed method for forming future mathematics teachers' methodological knowledge and skills is effective.

Among the calculated ten coefficients, the coefficient of manifestation of motivational and reflexive criteria was the lowest. The authors explain this, in particular, because students of the 1st and 2nd courses do not have much experience in assessing their activities, including forming methodological knowledge and skills. Most junior students are just getting used to self-esteem and reflection and usually do it in the teacher's direction. Among the students of the 1st and 2nd courses (the period of studying mathematical analysis), the indifferent attitude to the future profession prevails, motives of mastering methodological knowledge and skills are situational, and needs only partially realised. Similar results were obtained in the work of the authors [27].

Lectors of pedagogical and classical universities can use the results of this study during the preparation of the curriculum and work program, syllabus of mathematical analysis, the organisation of the classroom, independent and research work, etc., by specialists who teach mathematics teachers in training courses and by teachers and students.

CONCLUSIONS

The study identifies and characterises the methodological knowledge that should be formed during the training in mathematical analysis. It has offered the groups of methodological skills based on the specified methodological knowledge and indicated the methods of formation of future mathematics teachers' methodological knowledge and skills with verification of its efficiency.

The authors have established that the issue of the formation of methodological knowledge and skills is not considered separately from increasing the qualitative training of future mathematics teachers. The formation of methodological knowledge and skills cannot occur apart from the acquisition of subject knowledge and the formation of skills to apply them. Focusing students' attention on methodological knowledge and skills contributes to the systematisation of subject knowledge, and methodological knowledge is a means for systematically assimilating subject knowledge. At the teacher training level, both in mathematical disciplines and in general ones, the effectiveness of teacher work at school depends, among other things, on the formation of methodological knowledge and skills. The high level of methodological knowledge and skills allows students to master general knowledge and skills that will contribute to introducing STEM education elements. Further research can be performed in the following areas:

1) The formation of future specialists' methodological knowledge and skills in the field of Mathematics and Statistics (speciality 111 Mathematics);

2) Methodological knowledge and skills as a basis for building integrated courses for future teacher training in Natural Sciences in the context of the introduction of STEAM education.

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