Finite Element Analysis of Continuous Plates Using a High-Performance Programming Language (MATLAB)

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Abstract. This paper uses MATLAB, a finite element software program to compare the results of several finite analysis methods for continuous plates and check the degree of correlation with the exact values obtained by Timoshenko (1959) and Cheung (1996). The results showed little or no significant difference between plates in finite elements. Two different finite Numerical techniques are used. The Finite Strip and Exact methods and their results are compared to the results from the MATLAB program. Finite element analysis (FEA) workflow using MATLAB includes generation of meshes, geometry creation, defining physics of load, initial conditions and boundary problems, calculation, and results from visualization. FEA is a very general approach for solving Equations in science and engineering. This work offers solutions to the increasing errors associated with several other numerical methods when solving any equations of plates (continuous). It makes it easier to calculate and design larger structures through geometry discretization of plates and plains into more minor elements.

Keywords: Finite Element Analysis; Finite Element Method; MATLAB; Continuous Plates; Finite Strip Method; Exact Method.

INTRODUCTION

The finite element method (FEM) is a technique that is used to analyze finite elements of any given phenomenon. For any physical phenomena to be quantified and comprehended, it is necessary to use mathematics; such physical phenomena can include thermal transport, structural or fluid behaviour, biological cell growth, and wave propagation. Partial differential equations (PDEs) describe most of these processes. However, the finite element method and other numerical techniques developed over the past decades are used to solve partial differential equations using a computer.

Finite element analysis is a numerical technique based on the finite element method (FEM). This technique uses computer programs to predict behaviours of several physical systems, such as the deformation of solids, fluid flow, and heat conduction. Engineers and physicists use FEA as it allows the application of physical laws to practical scenarios. This allows for precision, versatility, and practicality.

FEM is widely used to get solutions for complex problems in the engineering field that are formulated using PDEs.

The development of reliable computer programs is required for numerical simulations based FEM to get correct solutions. This undergraduate thesis aims to help students understand how finite element processes solve problems through computer programs. The foundations of FEM and other computational problems have been taught and discussed in class lectures. Although, a class alone is not sufficient to grasp deep knowledge of how the operations like matrix assembly, boundary conditions, etc. all work effectively.

The role of plate structures in every structural system that man has ever built cannot be over-emphasized. Closed-form solutions to specific plate problems are seen in the literature and are an asset to this century’s designers.
These solutions, however, are generally for plates with relatively simple support and loading conditions and restricted to elastic, homogenous, isotropic plates of constant thickness. During the early periods of engineering, plates were designed with much more strength than was required. With the introduction of digital computers, investigators have all focused on numerical techniques and their study, like the FEA, used to solve the problem of plate structures.

This work simply tries to offer solutions linked to the other numerical methods in solving continuous plate’s equations through FEA. It aims at:

1. Reducing errors in calculation.
2. Aiding calculations of larger structures.
3. Aiding in the design of larger structures.
4. Develop programs that tackle issues on any form of continuous plates.

**METHODOLOGY**

Matrix Laboratory is a programming software used for quick and easy scientific calculations and Input/Output (I/O).

It has hundreds of built-in functions for a wide range of computations and many toolboxes set for different research disciplines, for example, data analysis, optimization, and statistics. Among much strength:

1) MATLAB may behave as a calculator or as a programming language.
2) MATLAB combines calculations and efficiently plots graphs.
3) MATLAB is not challenging to learn.
4) MATLAB is interpreted (not compiled), and errors are not hard to fix.
5) MATLAB is optimized to perform matrix operations quickly and efficiently.
6) MATLAB has some object-oriented elements.

**Finite element analysis of continuous plates. Outline of steps.** In any region of space where a particular phenomenon is occurring (continuum), a problem of any dimension, the field variables (stress, displacement, etc.) can assume an infinite set of values because it is a function of each generic point on the continuum. The finite element discretization procedure decreases the issues to one with a limited number of unknowns by dividing the solution region into smaller bits and expressing the unknown field variable in terms of an appropriate approximating function within each element. The values of the field variables gotten for specific points which let on the element boundaries every other point in which the details are completely defined using the interpolation (approximating) function.

**Step 1. Discretization of plate.** The discretization is done accordingly, and if the number of divisions increases, it is done similarly. Sixty-four elements were discretized. The node and element numbers are also shown below [35].

Figure 1 shows the figures denoting the number of nodes in which the graphs are plotted in y and x directions using MATLAB for a 64-element division.

![Figure 1](image1.png)

Figure 2 shows the node numbering done in the MATLAB program for the x-axis for 64 elements. The same procedure is used to number nodes. When the node number increases, the total will start from the centre of the plate along the x axis from left to right.

Figure 3 shows the numbering of nodes done in the MATLAB program for the y-axis for 64 elements. The same procedure will follow in the number of factors; increased node numbering will be done along the centre line of the plate from top to bottom.

The first thing to do in FEA is to divide the rectangular plate into more minor elements. The discretization can be achieved using a variety of factors, which depends on the degree of accuracy needed and the nature of the solution region.
During the analysis of plates by FEM, the choice of the number and the kind of element used is only a matter of good judgment and experience. The number of finite elements in a portion of the solution region where the variable is being modelled (displacement) differs. An essential part of the discretization process is node identification and numbering. And the total of nodes should be orderly. This is important because the finite element equation generated for each element takes cognizance of the node numbers around that element.

**Step 2. Develop or write the MATLAB program for the plate.**

```matlab
% FiniteElementParkings1.m Program for Finite Element Analysis of
% Rectangular Plates subjected to Bending using
% Automatic Mesh Generation
% Data Programs
% FiniteElemMeshGen5a
% FiniteElemMeshGen5b
% FiniteElemMeshGen7a
% FiniteElemMeshGen7b
% FiniteElemMeshGen7c
% FiniteElemMeshGen7d

tic

% Ex = E;
% Ey = E;
% Vx = v;
% Vy = v;
% Gxy = E/(2*(1+v));
% Dx = Ex*t^3/(12*(1-Vx*Vy));
% Dy = Ey*t^3/(12*(1-Vx*Vy));
% D1 = Vx*Ey*t^3/(12*(1-Vx*Vy));
% Dxy = Gxy*t^3/12;

Ks=zeros(3*f,3*f);
K=zeros(12,12,s);
Ks=zeros(3*f,3*f);
K=zeros(12,12,s);

% ELEMENT STIFFNESS MATRIX CALCULATION

disp ('ELEMENT STIFFNESS MATRIX CALCULATION:');
for a1=1:s
    Ks1 = [s1 -s2 0 s7 0 -s8
          s2 s5 0 s9 s10
          -s2 s3 s5 0 s9 s10
          -s4 s5 s6 s8 s10 s11
          s7 0 s8 s1 s2 s4
    ];
end
```
% \begin{align*}
  k(:,:,a1) &= \begin{bmatrix} k1 & k2 \\ k2' & k3 \end{bmatrix} * (15*a*b)^(-1); \\
  K(:,:,a1) &= \begin{bmatrix} k1 & k2 \\ k2' & k3 \end{bmatrix} / (15*a*b); \\
  \% \quad K = K(:,:,a1);
\end{align*}

\% \begin{align*}
  N1 &= \text{Connectivity}(a1,1) \\
  N2 &= \text{Connectivity}(a1,2) \\
  N3 &= \text{Connectivity}(a1,3) \\
  N4 &= \text{Connectivity}(a1,4) \\
  X &= \begin{bmatrix} 3*N1 & 3*N1 & 3*N1 & 3*N2 & 3*N2 & 3*N2 \\ 3*N3 & 3*N3 & 3*N3 & 3*N4 & 3*N4 & 3*N4 \end{bmatrix}; \\
  \% \quad Ks(X,X) = Ks(X,X) + K(:,:,a1); \\
  \% \quad Ks(X,X) = Ks(X,X) + K; \\
\end{align*}

\% \begin{align*}
  k(:,:,a1) &= \begin{bmatrix} k1 & k2' \\ k2' & k3' \end{bmatrix}; \\
  K(:,:,a1) &= \begin{bmatrix} k1 & k2' \\ k2' & k3' \end{bmatrix}/(15*a*b); \\
  \% \quad K = K(:,:,a1);
\end{align*}

\% \begin{align*}
  q = \text{SpecificGravity} \ast t; \\
  q &= 6; \quad \text{for example}
\end{align*}

\% \begin{align*}
  N1 &= \text{Connectivity}(sq(c3),1) \\
  N2 &= \text{Connectivity}(sq(c3),2) \\
  N3 &= \text{Connectivity}(sq(c3),3) \\
  N4 &= \text{Connectivity}(sq(c3),4) \\
  Mx1 &= a*b^2*q/24; \\
  My1 &= a^2*b*q/24; \\
  Fw1 &= a*b*q/4; \\
  Mx2 &= a*b^2*q/24; \\
  My2 &= a^2*b*q/24; \\
  Fw2 &= a*b*q/4; \\
  Mx3 &= a*b^2*q/24; \\
  My3 &= a^2*b*q/24; \\
  Fw3 &= a*b*q/4; \\
  Mx4 &= a*b^2*q/24; \\
  My4 &= a^2*b*q/24; \\
  Fw4 &= a*b*q/4; \\
  Nq &= \begin{bmatrix} Mx1 & My1 & Fw1 & Mx2 & My2 & Fw2 & Mx3 & My3 & Fw3 & Mx4 & My4 & Fw4 \end{bmatrix}; \\
  \% \quad\text{disp('Element Load Vector due to Distributed Load q: ’);} \\
  \% \quad\text{disp(Nq);} \\
\end{align*}
for c8 = 1:Pannels
   N22 = FirstCornerElemNumber(c8);
   N1 = Connectivity(N22,1);
   N2 = Connectivity(N22,2);
   N3 = Connectivity(N22,3);
   N4 = Connectivity(N22,4);
   X3 = [3*N1:2 3*N1-1 3*N1 3*N2-2 3*N2-1 3*N2 3*N3:2
        3*N3-1 3*N3 3*N4-2 3*N4-1 3*N4];
   ElemDispl = Ds(X3,1);
   x1 = 0;
   y1 = 0;
   B = subs(subs(B1,x1),y1); % B is the Strain Matrix
   Stresses = (D*B*Ds(X3,1));
   fprintf(‘ELEMENT NUMBER %d \n’,N22);
   NodalDisplacements = [ElemDispl(1) ElemDispl(2)
                        ElemDispl(3)];
   fprintf(‘Stresses at Node 1 of Panel %d \n’, c8);
   disp(‘Mx, My, Mxy : ‘);
   disp(Stresses);
   fprintf(‘Displacements at Node 1 of Panel %d \n’, c8);
   disp(‘Rotation about X, Rotation about Y, Deflection : ‘);
   disp(NodalDisplacements);
end

% Special Response
% Twisting and Bending Moments at Node 1 of First
% Corners of Pannels

Step 3. Run program for result. After all the essential
information, click on the run menu to run the
program for the result.

Step 4. Compare the result with the finite strip
method and the exact method. After getting the
results, compare the result with Timoshenko’s
exact method [40] and Cheung’s finite strip
method [4].

Step 5. Solve for a plate with different widths and
lengths. Assume the width of a plate is four and
the length is also four for each of the spans. Assume
also the value of q in the UDL to be 10000 N/m²,
with thickness as 0.2; Young’s modulus E = 30×10⁶ N/m², Poison’s ratio ν = 0.2 and
D = Et³/(12(1–ν²)).

RESULTS AND DISCUSSION

Finite element analysis for thin plates continuous
over three spans. This section of the research
work has solved a thin rectangular plate continuous
over three spans using the limited element
software program developed by the authors. The
scale is simply supported on all edges, including
the intermediate lines.

Plate properties: S – Number of finite element;
W – Width of plate; a=1; L – Length of plate = 3a = 1;
E – Young’s modulus = 30×10⁶ N/m²; V – Pois-
son’s ratio ν = 0.2; t – thickness = 0.2; q – Uniformly
distributed load (UDL) = 10000 N/m².
The analysis has been done for: a plate with width and length equals one; a plate with width and length equals four with the same thickness.

\[ D = \frac{E t^3}{12(1 - \nu^2)} = \frac{30 \times 10^6 \times 0.2^3}{12(1 - 0.2^2)} = 20833.33 \text{ N/m}^2 \]

Results for a plate with width and length equal to one (Table 1). The results from this research were compared to [40, p. 231; 4, pp. 87–88].

Result for plate with width and length equals to four (Table 2). Consider a continuous plate with three equal spans of width = 4 and length = 4 using the same thickness of 0.2; \( q = 10000 \text{ N/m}^2; D = 20833.33 \text{ N/m}^2; a = 4 \).

### Table 1 – Refined Methods – Finite element answer, and the answers also corresponds with that of finite strip method and exact method

<table>
<thead>
<tr>
<th>m/n</th>
<th>Number of finite elements</th>
<th>Deflection of the middle of the panel 1, WA (( \frac{qa4}{D} ))</th>
<th>Deflection of middle of panel 2, W</th>
<th>Longitudinal moment of first interior support, ( M_y B ) (( qa^2 ))</th>
<th>Transverse moment of panel 2, ( M_x C )</th>
<th>Longitudinal moment of panel 2, ( M_y C )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>48</td>
<td>-0.0719</td>
<td>0.0036</td>
<td>-6112</td>
<td>5152</td>
<td>6064</td>
</tr>
<tr>
<td>7</td>
<td>108</td>
<td>-0.0719</td>
<td>0.0036</td>
<td>-6112</td>
<td>5152</td>
<td>6064</td>
</tr>
<tr>
<td>9</td>
<td>192</td>
<td>-0.0719</td>
<td>0.0036</td>
<td>-6112</td>
<td>5152</td>
<td>6064</td>
</tr>
<tr>
<td>11</td>
<td>300</td>
<td>-0.0720</td>
<td>0.0036</td>
<td>-6112</td>
<td>5152</td>
<td>6096</td>
</tr>
<tr>
<td>13</td>
<td>432</td>
<td>-0.0719</td>
<td>0.0036</td>
<td>-6112</td>
<td>5152</td>
<td>6064</td>
</tr>
<tr>
<td>15</td>
<td>588</td>
<td>-0.0718</td>
<td>0.0036</td>
<td>-6112</td>
<td>5136</td>
<td>6048</td>
</tr>
</tbody>
</table>

### Table 2 – Finite element method for the continuous plate

<table>
<thead>
<tr>
<th>m/n</th>
<th>Number of finite elements</th>
<th>Deflection of the middle of the panel 1, WA (( \frac{qa4}{D} ))</th>
<th>Deflection of middle of panel 2, W</th>
<th>Longitudinal moment of first interior support, ( M_y B ) (( qa^2 ))</th>
<th>Transverse moment of panel 2, ( M_x C )</th>
<th>Longitudinal moment of panel 2, ( M_y C )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>48</td>
<td>-0.0585</td>
<td>0.0029</td>
<td>-0.0382</td>
<td>0.0322</td>
<td>0.0379</td>
</tr>
<tr>
<td>7</td>
<td>108</td>
<td>-0.0585</td>
<td>0.0029</td>
<td>-0.0382</td>
<td>0.0322</td>
<td>0.0379</td>
</tr>
<tr>
<td>9</td>
<td>192</td>
<td>-0.0586</td>
<td>0.0029</td>
<td>-0.0382</td>
<td>0.0323</td>
<td>0.0381</td>
</tr>
<tr>
<td>11</td>
<td>300</td>
<td>-0.0585</td>
<td>0.0029</td>
<td>-0.0382</td>
<td>0.0322</td>
<td>0.0379</td>
</tr>
<tr>
<td>13</td>
<td>432</td>
<td>-0.0585</td>
<td>0.0029</td>
<td>-0.0382</td>
<td>0.0322</td>
<td>0.0379</td>
</tr>
<tr>
<td>15</td>
<td>588</td>
<td>-0.0584</td>
<td>0.0029</td>
<td>-0.0372</td>
<td>0.0317</td>
<td>0.0378</td>
</tr>
<tr>
<td>Finite strip method [4]</td>
<td>-0.0583</td>
<td>0.0029</td>
<td>-0.0372</td>
<td>0.0319</td>
<td>0.0373</td>
<td></td>
</tr>
<tr>
<td>Exact method [40]</td>
<td>-0.0583</td>
<td>0.0029</td>
<td>-0.0381</td>
<td>0.0317</td>
<td>0.0375</td>
<td></td>
</tr>
</tbody>
</table>

### CONCLUSIONS

The results from MATLAB for continuous plates exhibit a high degree of correlation with the values of [40] and [4]. The analysis shows that the accuracy of results obtained for the plate in various finite elements had negligible differences.

Computerization of plate analysis by numerical methods (finite element method) presents the difficulty of computer memory management and ample execution time. It was observed that the execution time for various element software would increase as the number of finite elements. A new area of research has arisen recently, which would help tackle the problem of high execution time and efficient memory usage during program execution. Given the trouble of execution time, it is advised that a computer of higher memory capacity be used at least a quad-processor (4 GB RAM and above) for calculations to generate the desired solution.
REFERENCES


